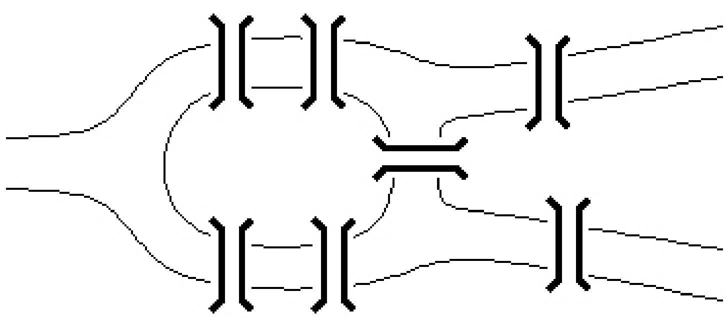
MATH 145 Final Exam 14 December 2005

Name: ____

Do any 6 of the following problems (100 points total).

Please circle the problems you want to be graded.
All statements must be proved. Answers without proofs may receive 0 credit.

1. Prove that if 40 coins are distributed among 9 bags so that each bag contains at least one coin, then at least two bags contain the same number of coins. Is your proof direct, by contradiction, or by contrapositive?
2. Find a formula for $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \ldots + \frac{1}{(2n-1)(2n+1)}$.
3. December 14, 2005 is a Wednesday. What day of the week is December 14, 2025?
4. Solve the inequality: $x^2 - |7x+15| \geq 3$.
5. The number 8^{2005} is written on a blackboard (it contains over 1800 digits, so I won't write it out here). The sum of its digits is calculated, then the sum of the digits of the result is calculated and so on, until we get a single digit. What is this digit?
6. A box contains 300 matches. Players take turns removing no more than half the matches in the box. The player who cannot take any match(es) loses. Find a winning strategy for one of the players.
7. Below is a plan of Konigsberg. As discussed in class, it is not possible to design a tour of the town that crosses each bridge exactly once and returns to the starting point. Could the citizens of Konigsberg find such a tour by building a new bridge?



8. Evaluate the integral: $\int_0^1 \arcsin(x)dx$ (hint: use areas).

For extra credit (15 points):

- Is it possible for a chess knight to pass through all the squares of a 4×2005 board having visited each square exactly once, and return to the initial square?