Practice Test 1

Note: The actual test will contain 4 problems, and you will need to choose and do any 3 of them.

1. Prove that for any natural n,
$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$
2. Let $\{F_0,\ F_1,\ F_2,\ \ldots\}$ be the Fibonacci sequence defined by $F_0 = 0$, $F_1 = 1$, and $F_{n+1} = F_n + F_{n-1}$, $n \geq 1$. Prove that $F_{n-1}^2 + F_n^2 = F_{2n-1}$
3. Kevin is paid every other week on Friday. Show that every year, in some month he is paid three times.
4. Let f be a one-to-one function from $X = \{1, 2, 3, 4, 5\}$ onto X. Let $f^k = \underbrace{f \circ f \circ \cdots \circ f}_{k\ times}$ denote the k-fold composition of f with itself. Show that for some positive integer m, $f^m(x) = x$ for all $x \in X$.
5. Six integer numbers, a_1, a_2, a_3, a_4, a_5, and a_6 are chosen randomly. Prove that $\prod\limits_{1 \leq i < j \leq 6} (a_i - a_j)$ is divisible by 10.
6. Show that $2^{457} + 3^{457}$ is divisible by 5.
7. Solve for x: $|x+1| + 5 - x^2 \geq 0$