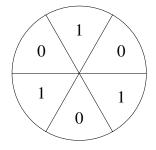
## Practice Test 2

- 1. Let  $F_0 = 0$ ,  $F_1 = 1$ ,  $F_2 = 1$ , ...,  $F_{99}$  be the first 100 Fibonacci numbers (recall that  $F_n = F_{n-1} + F_{n-2}$  for  $n \ge 2$ ). How many of them are even?
- 2. Two players play the following game. Turns alternate. At each turn, a player removes 1, 2, or 4 coins from a pile that initially had 10 coins. The game ends when all coins have been removed. The player who cannot make a move loses. Find a winning strategy for one of the players.
- 3. A circle is divided into six sectors. Then the numbers 1, 0, 1, 0, 1, 0 are written into the sectors as shown below. We may increase any two neighboring numbers by 1. We may repeat this step as many times as we want. Is it possible to equalize all the numbers?



4. Prove that a  $6 \times 6$  board cannot be covered by L-tetrominoes.

## Extra credit:

• Is it possible for a chess knight to pass through all the squares of a 4 × 2021 board having visited each square exactly once, and return to the initial square?