Review Problems

- 1. Let S be a set of 25 points such that, in any 3-subset of S, there are at least two points with distance less than 1. Prove that there exists a 13-subset of S which can be covered by a disk of radius 1.
- 2. Prove that if p > 3 is prime, then $p^2 \equiv 1 \pmod{24}$.
- 3. We strike the first digit of the number 7¹²³⁴, and add it to the remaining number. This is repeated until a number with ten digits remains. Prove that this number has two equal digits.
- 4. Find $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{n-1}{n!}$.
- 5. There are eight people in a room. Every person counted how many people he knows. (Assume that if A knows B then B knows A.)
 - (a) The numbers are 0, 1, 1, 2, 2, 3, 4, 4. Prove that somebody made a mistake.
 - (b) Can the numbers be 0, 1, 2, 3, 4, 5, 6, 7?
- 6. Show that it is not possible to cover any rectangle by one tile of type 1 shown below, one tile of type 3, and any number of tiles of type 2.

