

Study Guide and Sample Problems for Test 2

Note: the actual test will consist of five or six problems, some of which will be computational, some will ask for a brief explanation, and some will require a rigorous detailed proof. Some of the problems will be very similar to homework problems and/or those discussed in class, but some will be different. So make sure that you understand well all the concepts discussed, know precise definitions and basic properties, rather than memorize how to solve specific problems.

1. Binomial theorem and probability

- Explain why there are $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ ways to choose k objects out of n .
- Expand: $(x + y)^n$.
- Find the first three terms in the expansions of $(2x + 1)^5$, $(x - 2y)^{10}$.
- Prove that in Pascal's triangle, each number is equal to the sum of the two numbers above it.
- How many ways are there to choose 3 cards from a deck of 52 cards? How many ways are there to choose 3 cards from the 12 "face" cards (J, Q, K)? If three cards are chosen randomly for a deck of 52 cards, what is the probability that all three are face cards?

2. Polynomials

- What is a polynomial?
- Prove that any polynomial of an odd degree has at least one real root.
- How many real roots can a polynomial of fifth degree have? Give an example for each case.
- How many real roots can a polynomial of sixth degree have? Give an example for each case.
- Prove that if r is a root of a polynomial $p(x)$, then $p(x) = (x - r)q(x)$ for some polynomial $q(x)$.

3. Functions

- What is a function?
- List at least 4 ways to define/represent a function.
- Give the definitions of: domain, range, intercepts, asymptotes, roots, graph.
- First sketch the graphs of $\sin(x)$ and $\cos(x)$. Then use these to sketch the graphs of $\tan(x)$, $\cot(x)$, $\sec(x)$, and $\csc(x)$.
- Sketch the graphs of a^x and $\log_a x$ (consider all necessary cases for the base a).

4. Equations and inequalities

- Derive or prove the quadratic formula.
- What does it mean to solve an equation or an inequality?
- Is guess and check a good method for solving equations? Explain, provide examples to support your arguments.
- Solve: $\sqrt{(x-2)(x-7)} = 5\sqrt{2}$.
- Solve: $\log_{2x-3} 9 = 2$.
- Solve: $\tan^3 x - \tan^2 x - 3 \tan x + 3 = 0$.
- Solve: $x^2 < 15$; $x^2 > 15$; $x^2 \leq 15$; $x^2 \geq 15$; $12 < x^2 < 15$.
- Solve: $x^{x+1} = 8$
- How many integer solutions does the following system of inequalities have?

$$x < 2y, \quad x + y > 3, \quad y < 4$$

5. Proportional reasoning, ratios, and rates

- If $5x = 6y$, what is the ratio of x to y ?
- The distance fallen by a freely falling body is proportional to the square of the time it has been falling. If an object falls 144 feet in 3 seconds, how far will it fall in 6 seconds?
- If Misha can eat a large box of oatmeal in two months and Katya can eat the same box in one month, in how much time will they eat one box together?
- Luke can solve 5 problems in 10 minutes. How many problems can he solve in 30 minutes? Five hours? One week?