MATH 149

Spring 2009

Final Exam

Name: _____

- No books, notes, or calculators are allowed.

- Please show all your work. You can use the back of each page for scratch paper.

- Please explain all your solutions and answers. Answers without reason will not receive full credit (if any).

1. (10%) Two triangles are similar. One triangle has sides 4, 5, and 6. Two sides of the other triangle are 10 and 15. What is the third side of the second triangle?

2. (10%) Find the domain of the function $f(x) = \sqrt{x^2 - 2x - 15}$.

3. (10%) Explain how an approximation for the number π can be computed.

4. (10%) A Math Club is designing a t-shirt. They want the t-shirt (i.e. the background) to be dark: dark gray, dark blue, or black. There are five options (white, yellow, light blue, red, green) for the color of the text and the same five options for the color of graphics. However, they want the text and graphics to be different colors. How many different designs are possible?

5. (12%) Prove the Pythagorean Theorem.

6. (16%) Define a relation on $\mathbb{N} \times \mathbb{N}$ by $(a, b) \sim (c, d)$ if $a + d = b + c$.

(a) Show that this is an equivalence relation.

(b) Describe the equivalence class of $(2, 5)$. (In particular, say how many elements it contains. List a few elements.)

7. (16%)

  (a) Explain how you could estimate the height of a tall tree. List any materials and/or conditions necessary. Which concepts or theorems did you use?

  (b) Describe any other practical application or project that uses high school geometry material.

8. (16%) Determine which of the following statements are true. Prove your claims.

   (a) The sum of two positive irrational numbers is always irrational.

   (b) The sum of a positive irrational and a positive rational number is always irrational.

9. Optional (for extra credit, 10%): What is the volume of the ellipsoid with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$? (Explain!)