MATH 149

Spring 2009

Test 1

Name: _____

- No books, notes, or calculators are allowed.
- Please show all your work. You can use the back of each page for scratch paper.

1. (15%) Which of the following sets with the given operations are groups? For those that are not, give at least one axiom that fails. (For those that are groups, it is sufficient to say that they are.)

   (a) $\mathbb{Z}$ with subtraction

   (b) $\mathbb{Q}$ with addition

   (c) $\mathbb{R}$ with multiplication

2. (20%) Define a relation on $\mathbb{R}$ by $a \sim b$ if $a - b \in \mathbb{Q}$.

   (a) Show that $\sim$ is an equivalence relation.

   (b) What is the equivalence class of 1?

3. (15%) Prove that a natural number is divisible by 9 if and only if the sum of its digits is divisible by 9.

4. (15%) Find the multiplicative inverse of the element $5 + 4\sqrt{2}$ in $\mathbb{Q}(\sqrt{2})$.

5. (15%) Explain why when a fraction is converted into a decimal, the resulting decimal is either terminating or periodic.

6. (20%) Find at least one complex solution to the equation $x^4 = -16$. Find both rectangular $(a + bi)$ and polar $(re^{i\theta})$ representations of your root.

7. Optional (for extra credit, 10%): Given a fraction $\frac{a}{b}$ in its reduced form, how can you tell (without using long division) whether it is represented by a terminating or infinite decimal? E.g., which of the following fractions are represented by terminating decimals and which are represented by infinite decimals: $\frac{11}{2500}$, $\frac{17}{125}$, $\frac{2}{15}$, $\frac{9}{40}$, $\frac{3}{7}$, $\frac{7}{12}$, $\frac{5}{8}$? Explain how you determined the answer (without actually converting these into decimals).