2014
LEAP FROG RELAY GRADES 11-12
PART I

No calculators allowed
Correct Answer $= 4$, **Incorrect Answer** $= -1$, **Blank** $= 0$

(1) Let r be the remainder of $1 + 2^2 + 3^3 + 4^4 + 5^5 + 6^6 + 7^7 + 8^8 + 9^9 + 10^{10}$ when divided by 3. Let s be the sum of the last digits of each of the terms of the sum above. What is $r + s$?

(a) 47
(b) 49
(c) 45
(d) 42
(e) None of the above

(2) Let

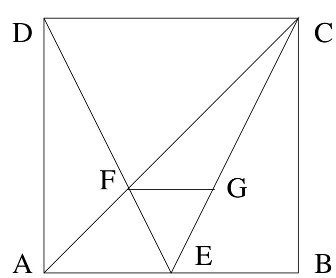
$$S = \sqrt{1} + \sqrt{1 + 2^3} + \sqrt{1 + 2^3 + 3^3} + \cdots + \sqrt{1 + 2^3 + 3^3 + \cdots + 2014^3}$$

Then,

(a) $S = \binom{2016}{3}$
(b) $S = \binom{2014}{3}$
(c) $S = \binom{2016}{6}$
(d) $S = \binom{2013}{6}$
(e) None of the above

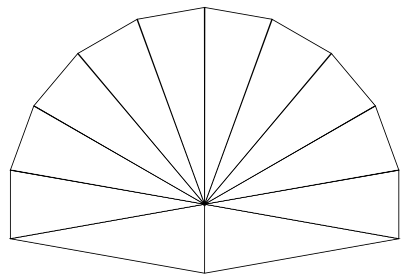
Where, $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$.

(3) The square $ABCD$ has sides of length 2. Point E is the midpoint of edge AB. Point F is the intersection of lines AC and DE. Line FG is parallel to line AB. The area of $\triangle EFG$ is:



(a) $\frac{2}{3}$
(b) $\frac{1}{3}$
(c) $\frac{2}{9}$
(d) $\frac{4}{9}$
(e) None of the above

(4) An isosceles triangle $\triangle ABC$ has equal angles $B = C$. Twelve copies of $\triangle ABC$ are arranged around a common vertex without gaps or overlaps as shown. (The common vertex is surrounded by 10 angles equal to A and 2 angles equal to B.) Find the measure of A in degrees.



(a) $15°$ (b) $25°$

(c) $18°$ (d) $20°$

(e) None of the above

(5) Let $S = \{1, 4, 9, 16, 25, ...\}$ be the set of squares of positive integers. Let $t \in S$ be such that $t - 76 \in S$. What is $76t$?

(a) $30,400$ (b) $27,436$

(c) $24,624$ (d) $33,516$

(e) None of the above

(6) Given that $2^{60} = 1,152,921,504,606,846,976$, find the first four digits (reading left to right) of 2^{61} and 2^{59}, then add these 8 digits up to get:

(a) 35 (b) 30

(c) 32 (d) 28

(e) None of the above

(7) Given that a and f are integers between 0 and 9 such that $a^5 + 1 = f \cdot 1111$, find $a + f$.

(a) 15 (b) 13

(c) 12 (d) 10

(e) None of the above

(8) Three solutions of the equation $m!(m+1)! = n!$ are $(m,n) = (0,0)$, $(m,n) = (0,1)$, and $(m,n) = (1,2)$. There is a unique fourth solution to this equation so that $0 \le n \le 10$ and $0 \le m \le 10$. For that solution, find $n - m$.

(a) 6
(b) 5
(c) 3
(d) 4
(e) None of the above

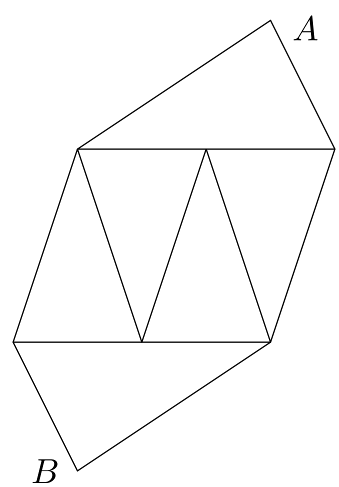
(9) Given that $2+\sqrt{3}$ is one of the solutions of the equation

$$x^4 - 14x^3 + 54x^2 - 62x + 13 = 0$$

how many complex solutions does this equation have?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

(10) The adjacent figure has six non-overlapping congruent isosceles triangles. In each triangle the equal sides are 2 units and the base is 1 unit. Find the distance from A to B.



(a) $\sqrt{19}$
(b) $\sqrt{17}$
(c) $3\sqrt{2}$
(d) $2\sqrt{5}$
(e) None of the above