2014
LEAP FROG RELAY GRADES 11-12
PART II

No calculators allowed
Correct Answer $= 4$**, Incorrect Answer** $= -1$**, Blank** $= 0$

(11) Let $f(x) = |3x - 2|$. Find the sum of *all* real solutions, x, to the equation $f(f(x)) = 2$.

(a) 2

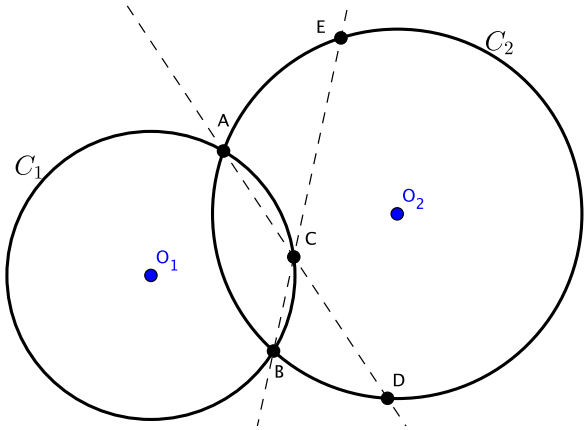
(b) $\frac{14}{9}$

(c) $\frac{16}{3}$

(d) 0

(e) None of the above

(12) Circles C_1 and C_2, centered at O_1 and O_2 respectively, intersect in A and B. Point C is on circle C_1, and AC, BC meet circle C_2 in D and E, respectively. Find the measure of the angle between $\overleftrightarrow{O_1C}$ and $\overleftrightarrow{DE}$.



(a) 92°

(b) 95°

(c) 90°

(d) 88°

(e) None of the above

(13) Find the sum of all of the real numbers x which satisfy

$$\sin x + \cos x = \sqrt{\frac{2+\sqrt{3}}{2}}$$

with $0 < x < \pi/2$.

(a) $\pi/2$

(b) $\pi/6$

(c) $2\pi/3$

(d) $\pi/6$

(e) None of the above

(14) Let $a > 1$ and $b > 1$ be real numbers such that

$$\log_{10}(a+b) = \log_{10} a + \log_{10} b$$

What can you say about the value of

$$\log_{10}(a-1) + \log_{10}(b-1)?$$

(a) 2 (b) 3

(c) 1 (d) 0

(e) None of the above

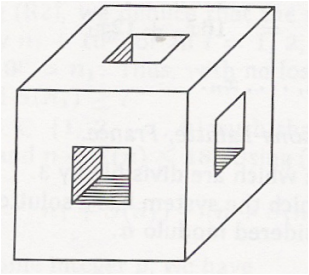
(15) A working crew of x men work x hours a day for x days to dig a tunnel of length x yards. A second crew of y men work y hours a day for y days. What length (in yards) of the continuation of the tunnel would you expect them to dig? You may assume all men work at the same rate.

(a) y (b) x^2/y^3

(c) y^3/x^2 (d) y^2/x^2

(e) None of the above

(16) In a cube of side 3 in, in the center of three different (and not opposite) faces we bore a square perforation of side 1 inch that goes across the cube as far as the opposite face. We thus obtain the following figure:



Determine the surface area of the resulting solid

(a) 72 in^2 (b) 70 in^2

(c) 68 in^2 (d) 74 in^2

(e) None of the above

(17) Simplify

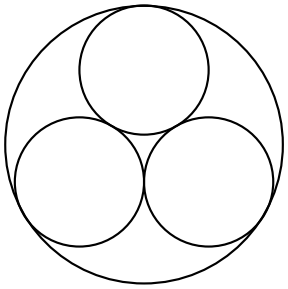
$$\sqrt{\frac{8^{10}+4^{10}}{8^4+4^{11}}}$$

(a) 64 (b) 32

(c) 16 (d) 8

(e) None of the above

(18) Given three distinct unit circles (i.e. circles of radius 1), each of which is tangent to the other two, find the radius of the circle which is tangent to all three circles and contains them.



(a) $\frac{2\sqrt{3}+1}{3}$ (b) $\frac{2\sqrt{3}}{3}$

(c) $2\sqrt{3}$ (d) $\sqrt{3}+1$

(e) None of the above

(19) Compute the integer k, $k > 2$, for which

$$\log_{10}\left[(k-2)!\right]+\log_{10}\left[(k-1)!\right]+2=2\log_{10}\left(k!\right).$$

(a) $k = 4$ (b) $k = 5$

(c) $k = 7$ (d) $k = 6$

(e) None of the above

(20) The smallest prime number that divides $2^{111}+3^{111}$ is

(a) 23 (b) $2^{111}+1$

(c) 17 (d) $3^{111}+1$

(e) None of the above