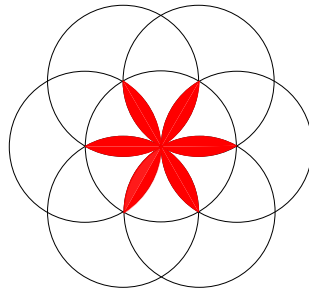


2013
Leap Frog Relay Grades 11-12
Part I (problems 1-10)

No calculators allowed
Correct Answer = 4 points
Incorrect Answer = -1 point
Blank = 0 points

1. Seven circles of radius 10 are arranged as in the figure. Note that the six outer circles all pass through the center of the inner circle, the inner circle passes through the center of each outer circle, and each outer circle passes through the center of the two outer circles it is adjacent to. The area of the shaded region is A . Which of the following is true about A ?



- (a) $40 \leq A < 60$ (b) $60 \leq A < 80$
- (c) $80 \leq A < 100$ (d) $100 \leq A < 120$
- (e) $120 \leq A < 140$
2. From the list of all natural numbers $2, 3, \dots, 999$, one deletes sublists of numbers nine times: At first, one deletes all even numbers, then all numbers divisible by 3, then all numbers divisible by 5, and so on, for the nine primes $2, 3, 5, 7, 11, 13, 17, 19, 23$. Find the sum of the composite numbers left in the remaining list.

- (a) 0 (b) 961
 (c) 3062 (d) 2701
 (e) 899

3. What are the solutions to

$$\tan\left(x + \frac{\pi}{6}\right) = \sqrt{3}$$

where $0 \leq x \leq 2\pi$

- (a) $\frac{11\pi}{6}, \frac{5\pi}{6}$ (b) $\frac{5\pi}{3}, \frac{2\pi}{3}$
 (c) $\frac{\pi}{3}, \frac{4\pi}{3}$ (d) $\frac{\pi}{6}, \frac{7\pi}{6}$
 (e) None of the above

4. Solve for x

$$\log_5(2x^2 - 3x + 1) - \log_5(x - 1) + \log_5 125 = 6$$

- (a) 1 (b) $\frac{1}{2}$
 (c) 5^3 (d) 6^3
 (e) None of the above

5. Given a parabola with directrix $y = -2$ and focus $(2, 2)$. What are the value(s) for m for which the line $y = mx$ is tangent to the parabola?

- (a) ± 4 (b) 0
 (c) 2 and 4 (d) -2 and -4
 (e) 0 and -8

6. Consider the equation

$$\sqrt{x + 3 - 4\sqrt{x - 1}} + \sqrt{x + 8 - 6\sqrt{x - 1}} = 1$$

where x represents a real number. How many solutions are there?

- (a) Exactly one solution (b) Exactly two solutions
 (c) Exactly three solutions (d) Exactly four solutions
 (e) Infinitely many solutions

7. The points A, B, C , and D are the vertices of a unit square. How many squares (including $ABCD$ itself) in the same plane have two or more of these points as vertices?
- (a) 13 (b) 12
(c) 9 (d) 5
(e) 4
8. How many two-digit numbers are divisible by each of their digits?
- (a) 9 (b) 10
(c) 11 (d) 13
(e) 14
9. The Fibonacci numbers $1, 1, 2, 3, 5, 8, \dots$ form a sequence where each term, after the first two, is the sum of the two previous terms. How many of the first 1000 terms are even?
- (a) 222 (b) 333
(c) 499 (d) 500
(e) 501
10. One sphere is inscribed in a cube, while the cube is also inscribed in another sphere. Find the ratio of the volumes of the larger sphere to the smaller sphere.
- (a) $\sqrt{3}$ (b) $2\sqrt{3}$
(c) $3\sqrt{3}$ (d) $3\sqrt{2}$
(e) None of the above