

MATH 149

Spring 2016

Test 2

Name: _____

- This test is take-home.
- Allowed time: 1 hour 15 min.
- Books, notes, calculators, etc. are allowed.
- Please show all your work.
- Turn in via email (mnogin@csufresno.edu) by 11:59 PM on April 13.

1. (15%) Explain, on a level accessible to an Algebra I student, why a decimal represents a rational number if and only if it is either terminating or repeating. (Your explanation may be informal, but should be complete/general. An example may be used to illustrate an idea, but just an example is not sufficient.)

2. (20%)

(a) Give an example of a set (of objects familiar to high school students) and an operation for which commutativity does not hold. (Provide a proof.)

(b) Does associativity hold for the above operation? (State a yes/no answer and provide a proof.)

3. (20%) For each of the following statement, determine whether it is true or false. (Provide a proof.)

(a) The sum of two positive irrational numbers is always irrational.

(b) The infinite decimal $0.123456789101112131415\dots$ (whose digits are those of all natural numbers) represents a rational number.

4. (15%) Compute: $(1 + i)^{20}$.

5. (30%) Which of the following statements are true, false, or incorrectly worded? Provide brief justifications for your answers. For those that are incorrectly worded, suggest a correction and determine their truth values. For those that are false, modify them to make true.

- The graph of a function $f : \mathbb{R} \rightarrow \mathbb{R}$ can have any number of horizontal asymptotes, from 0 to infinitely many.

- The point $(3, 4)$ lies on the graph of the function $x^2 + y^2 = 25$.

- Every function $f : \mathbb{R} \rightarrow \mathbb{R}$ has exactly one y -intercept.

- The pair $(-2, -4)$ is a solution of the function $f(x) = x^2 + 1$.

6. Optional (for extra credit, 10%):

Is the number $\frac{1}{20 + \sqrt{15}} + \frac{1}{20 - \sqrt{15}}$ rational or irrational?