2014 Leap Frog Relay Grades 9-10 Part I Solutions

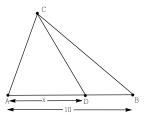
No calculators allowed Correct Answer = 4, Incorrect Answer = -1, Blank = 0

- 1. The sum of the prime divisors of 2014 is
 - (a) 76 (b) 78
 - (c) 80 (d) 82
 - (e) None of these

Solution. (e) Factor, $2014 = 2 \times 19 \times 53$. Since 2, 19 and 53 are primes, the sum of the prime divisors of 2014 is

$$2 + 19 + 53 = 74.$$

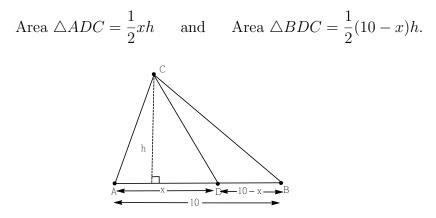
2. In $\triangle ABC$, the base length AB is 10 feet and the length AD is x feet. What is the value of x so that the area enclosed by $\triangle ADC$ is twice the area enclosed by $\triangle BDC$?



(a)
$$x = 8$$
.
(b) $x = \frac{9}{2}$
(c) $x = 5$
(d) $x = \frac{20}{3}$

(e) None of these

Solution. (d) Draw the altitude of $\triangle ADC$ whose length we'll call h. Note that h is also the altitude of $\triangle BDC$. Also note that the length of BD is 10 - x. So, we have



Then, the requirement that $\triangle ADC$ has twice the area of $\triangle BDC$ means

$$\frac{1}{2}xh = 2 \cdot \frac{1}{2}(10 - x)h.$$

Solve this equation for x to get $x = \frac{20}{3}$.

- 3. The Antarctic Shoe Store is running a 30% off sale. Sales tax is 10%. The pre-sale price of a pair of shoes is \$78. When you pay for your pair of shoes, you hand the cashier a \$100 bill. How much change do you get back?
 - (a) \$39.82 (b) \$39.86
 - (c) \$39.90 (d) \$39.94
 - (e) None of these

Solution. (d) Thirty percent off of 78 is $0.7 \times 78 = 54.60$. Add in the 10% sales tax, 54.60 + 5.46 = 60.06. The change you get back is \$100 - \$60.06 = \$39.94.

- 4. A cube of ice has melting so that its surface area has decreased by 19%. Assuming at all times, the cube maintains length = width = height, by what percentage has the volume decreased?
 - (a) 26.7% (b) 26.9%
 - (c) 27.1% (d) 27.3%
 - (e) None of these

Solution. (c) Denote the side length of the cube by x before melting and by y after melting. Then the two surfaces areas are $6x^2$ and $6y^2$, respectively. We are assuming $6y^2 = (1 - .19)6x^2 = .81 \times 6x^2$. This implies $y = \sqrt{.81}x = .9x$. Cubing each side gives us the volume, $y^3 = .729x^3$. So the volume has decreased by a factor of 1 - .729 = .271. This means, the volume has decreased by 27.1%.

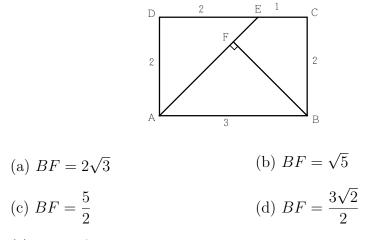
- 5. The 9-digit number N = 1234d4321 is divisible by 9. What is the value of the digit d?
 - (a) 7 (b) 6
 - (c) 5 (d) 4
 - (e) None of these

Solution. (a) A number is divisible by 9 precisely when its digits add up to a multiple of 9. The sum of the digits of N is

$$1 + 2 + 3 + 4 + d + 4 + 3 + 2 + 1 = 20 + d.$$

In order for 20 + d to be divisible by 9, we must have d = 7. And indeed, $123474321 = 9 \times 13719369$

6. In the figure below ABCD is a rectangle with indicated side lengths. Also, $\overline{BF} \perp \overline{AE}$. Find the length of BF.



(e) None of these

Solution. (d) Since $m \angle ABF = 90 - m \angle BAF = m \angle DAE$, we have similar triangles $\triangle BAF \sim \triangle AED$. Hence,

$$\frac{BF}{AB} = \frac{AD}{EA}.$$

Now, AB = 3, AD = 2 and by the Pythagorean Theorem applied to $\triangle ADE$ we have $EA = 2\sqrt{2}$. This gives us,

$$\frac{BF}{AB} = \frac{AD}{EA} \Longrightarrow \frac{BF}{3} = \frac{2}{2\sqrt{2}} \Longrightarrow BF = \frac{3\sqrt{2}}{2}.$$

7. Find the value of a so that the two lines 2013x + 2012y = 1 and 2014x + ay = 1 are mutually perpendicular.

(a) $-\frac{2027091}{1009}$	(b) $-\frac{2027091}{1008}$
(c) $-\frac{2027091}{1007}$	(d) $-\frac{2027091}{1006}$

(e) None of these

Solution. (d) The slopes of mutually perpendicular lines are negative reciprocals of one another. We find the slopes by solving for y in each

line.

Line₁ :
$$2013x + 2012y = 1 \Longrightarrow y = -\frac{2013}{2012}x + \frac{1}{2012}$$

Line₂ : $2014x + ay = 1 \Longrightarrow y = -\frac{2014}{a}x + \frac{1}{a}$

The respective slopes are then $m_1 = -2013/2012$ and $m_2 = -2014/a$. Since the lines are mutually perpendicular, we must have $m_1 = -1/m_2$,

$$-\frac{2013}{2012} = \frac{a}{2014}.$$

Solving for a, gives us

$$a = -\frac{2013 \times 2014}{2012}$$
$$= -\frac{2013 \times 1007}{1006}$$
$$= -\frac{2027091}{1006}.$$

- 8. A standard calendar year has 365 days. A leap year has 366 days. A year is a leap year if it is divisible by 4, *except* if it is a new century year not divisible by 400. So 1900 was not a leap year (1900 *is not* divisible by 400), but 2000 was a leap year (2000 *is* divisible by 400). December 25, 2013 was a Wednesday. What day of the week will December 25 be in the year 3013?
 - (a) Friday (b) Saturday
 - (c) Sunday (d) Monday
 - (e) None of these

Solution. (b) If we divide 365 by 7, we get a remainder of 1. This means that without leap years, the days of the week for a particular

calendar date would advance 1 day per year. There is 1000 years from 2013 to 3013, so this would account for 1000 added days. However, we must then add the 250 leap days (1000/4) less the non-leap days in the new century years not divisible by 400 (there are 8 of these), giving a total of 1000 + 250 - 8 = 1242. Divide 1242 by 7 to get a remainder of 3. This means December 25, 3013 will occur 3 days after Wednesday, which is Saturday.

9. Lenny asks Rodney to choose a number. Lenny then doubles Rodney's number and subtracts 6. Lenny repeats this procedure 2 more times (for a total of 3 times) using the result of the previous calculation as the starting number with each repeat. Lenny then notes that the end result is the number 2014. What was Rodney's starting number?

- (c) 255 (d) 256
- (e) None of these

Solution. (e) Let's let x be Rodney's original number. Then the result of the 3 repeated procedures is

$$2(2(2x-6)-6) - 6 = 8x - 42.$$

Solve the equation 8x - 42 = 2014 to get x = 257, none of the answer choices provided.

10. Lenny has 45 coins (nickels, dimes and quarters) in his purse that add up to \$4. The Magic Fairy instantly switches the respective numbers of nickels and quarters, doubling the amount of money in Lenny's purse, giving him \$8. How many quarters did Lenny originally have?

- (c) 6 (d) 5
- (e) None of these

Solution. (d) Let n, d and q be the original number of nickels, dimes and quarters in Lenny's purse. Since there are 45 coins, we have our first equation:

$$Eq_1: n + d + q = 45.$$

The dollar amount is \$4, so this gives us another equation 5n + 10d + 25q = 400. Divide by 5 to get equation number 2,

$$Eq_2: n+2d+5q = 80.$$

After, the switch, we have 25n + 10d + 5q = 800 and upon dividing by 5, we have

$$Eq_3: 5n + 2d + q = 160.$$

Solve the above three equations in 3 unknowns, to get n = 25, d = 15and q = 5. So, Lenny originally had 5 quarters.