

# CSU FRESNO MATHEMATICS FIELD DAY

MAD HATTER MARATHON 9-10  
PART I

April 26<sup>th</sup>, 2014

1. Evaluate the following:

$$4 - 2 + 8 \div 2 \times (4 + 2).$$

(a)  $2\frac{2}{3}$

(b)  $6\frac{1}{3}$

(c) 26

(d) 30

2. Find the coefficient of  $x^6y^3$  in  $(x - 2y)^9$ .

- (a) 672
- (b) 144
- (c) -144
- (d) -672

3. Solve for  $x$

$$|x^2 - 4| \leq 0.$$

- (a)  $-2 \leq x \leq 2$
- (b)  $x \leq -2$  or  $x \geq 2$
- (c)  $x = 2$  or  $x = -2$
- (d) There is no solution.

4. Put the following geometric figures in order of decreasing perimeter.

I. a circle of radius 2.

II. a square of side length 4.

III. a right triangle with two sides of length 3 and 4.

(a) I., II., III.

(b) II., I., III.

(c) III., II., I.

(d) III., I., II.

5. What is  $\sqrt{33}$  to the nearest hundredth?

- (a) 5.74
- (b) 5.79
- (c) 5.83
- (d) 5.91

6. Determine the sum  $7 + 11 + 15 + \cdots + 83$ .

(a) 903

(b) 900

(c) 820

(d) 630

7. Find the first of three consecutive odd integers whose sum is 57.

- (a) 13
- (b) 15
- (c) 17
- (d) 19



8. The midpoint of segment  $\overline{PQ}$  is  $F$ , and the length of  $\overline{FQ}$  is  $3m$ .  
What is the length of  $\overline{PQ}$ ?

- (a)  $\frac{3}{2}m$
- (b)  $3m$
- (c)  $6m$
- (d)  $9m$

9. If  $a^x = 4$  and  $a^y = 64$ , what is the value of  $a^{x-y}$ ?

(a) -60

(b)  $\frac{1}{16}$

(c)  $\frac{1}{4}$

(d) 16

10. If 60% of Sam's jigsaw puzzles have 500 pieces each and 40% of his jigsaw puzzles have 300 pieces each, what is the average number of pieces per puzzle?

- (a) 360
- (b) 380
- (c) 400
- (d) 420

11. Determine the equation of the line that is a perpendicular bisector of the segment from  $(0, 4)$  to  $(4, 10)$ .

(a)  $y = -\frac{2}{3}x + \frac{25}{3}$

(b)  $y = -\frac{2}{3}x + 4$

(c)  $y = \frac{3}{2}x + 4$

(d)  $y = \frac{3}{2}x$

12. When a gymnast competes at the Olympics, each of six judges awards a score between 0 and 10. The highest and lowest scores are discarded, and the gymnast's final mark is the average of the remaining scores. What would a gymnast's mark be if the judges' scores were 9.6, 9.4, 9.5, 9.7, 9.2, and 9.6?

- (a) 9.5
- (b) 9.525
- (c) 9.55
- (d) 9.6

13. A dealer in rare metals owns 1,000 ounces of silver. Every year, she sells half of the silver she owns and does not acquire any more. How many ounces of silver will she own  $t$  years from now, where  $t$  is a positive integer?

- (a)  $\frac{1000}{2t}$  ounces
- (b)  $1000 \cdot 2t$  ounces
- (c)  $1000 \cdot 2^{-t}$  ounces
- (d)  $1000 \cdot 2^t$  ounces

14. For how many different positive integers  $n$  does  $\sqrt{n}$  differ from  $\sqrt{81}$  by less than 1?

- (a) 18
- (b) 34
- (c) 35
- (d) 36

15. When the price of gold went up, a jeweler raised the prices on some of his rings by 60%. On one ring, however, the price was accidentally reduced by 60%. By what percent must the incorrect price be increased to reflect the correct new price?

- (a) 60%
- (b) 120%
- (c) 300%
- (d) 400%



16. A right circular cylinder has a radius of 8 and height of  $\pi^2$ . If a cube has the same volume as the cylinder, what is the length of an edge of the cube?

- (a)  $4\sqrt{\pi}$
- (b)  $8\sqrt{\pi}$
- (c)  $4\pi\sqrt{\pi}$
- (d)  $4\pi$

17. For how many integers  $n$  is  $n^2 - 30$  negative?

(a) 5

(b) 6

(c) 11

(d) infinitely many

18. Because her test turned out to be more difficult than she intended it to be, a teacher decided to adjust the grades by deducting only half the number of points a student missed. (So, for example, if a student missed 10 points, she received a 95 instead of a 90.) Before the grades were adjusted, Maria's grade on the test was  $g$ . What was her grade after the adjustment?

(a)  $50 + \frac{g}{2}$

(b)  $\frac{50 + g}{2}$

(c)  $100 - \frac{g}{2}$

(d)  $\frac{100 - g}{2}$

19. If  $a$  and  $b$  are the lengths of the legs of a right triangle whose hypotenuse is 10 and whose area is 20, find  $(a + b)^2$ .

(a) 180

(b) 140

(c) 120

(d) 100

20. A jeweler has a 20 gram ring that is 60% gold and 40% silver. He wants to melt it down and add enough gold to make it 80% gold. How many grams of gold should be added?

- (a) 20 grams
- (b) 16 grams
- (c) 12 grams
- (d) 8 grams

21. Suppose  $a$  is an irrational number. Which of the following must also be an irrational number?

- (a)  $a^3$
- (b)  $\sqrt{|a|}$
- (c)  $\ln |a|$
- (d) None of these.

22. Determine the fraction represented by  $0.141414 \dots$

(a)  $\frac{14}{99}$

(b)  $\frac{7}{55}$

(c)  $\frac{14}{111}$

(d)  $\frac{14}{93}$

23. Let  $f(x) = x^6 + 7x^3 - 8$  and  $g(x) = (x^2 + x - 2)(x^2 + x + 1)$ . Suppose  $h(x)$  is a polynomial such that  $f(x) = h(x)g(x)$ . Determine  $h(1)$ .

- (a) 0
- (b) 1
- (c) 2
- (d) 3



24. An equilateral triangle and a regular hexagon have equal perimeters. What is the ratio of the area of the triangle to the area of the hexagon?

(a)  $\frac{2}{3}$

(b)  $\frac{3}{2}$

(c)  $\frac{1}{2}$

(d)  $\frac{2}{1}$

25. The tortoise and the hare are going to race again, but this time the hare gives the tortoise a 990 yard head start. If the hare can cover 10 yards each second, while the tortoise only covers 1 yard in 10 seconds, how long will it take the hare to catch the turtle?

- (a) 1 minute 20 seconds
- (b) 1 minute 40 seconds
- (c) 2 minutes
- (d) 2 minutes 3.75 seconds

26. Square  $ABCD$  has side length 10. If point  $E$  is on  $\overline{BC}$ , and the area of  $\triangle ABE$  is 40, what is  $BE$ ?

- (a) 4
- (b) 5
- (c) 6
- (d) 8

27. A lab has 55 rabbits, some of which are white and the rest of which are brown. Which of the following could represent the ratio of white rabbits to brown rabbits in the lab?

(a) 3:8

(b) 5:11

(c) 3:4

(d) 3:1

28. If 1 bloop = 7 blops and 2 blops = 1 blip, how many bloops are equal to 28 blips?

- (a) 2
- (b) 8
- (c) 14
- (d) 28

29. The sum of two positive numbers  $x$  and  $z$  is 50. If  $x$  is at most 10 more than 4 times  $z$ , which of the following includes the entire range of values for  $z$ ?

(a)  $0 \leq z \leq 4$

(b)  $0 \leq z \leq 8$

(c)  $8 \leq z \leq 10$

(d)  $8 \leq z \leq 50$

30. Steve ran a 12-mile race at an average speed of 8 miles per hour. If Adam ran the same race at an average speed of 6 miles per hour, how many minutes longer did Adam take to complete the race than Steve?

- (a) 9
- (b) 12
- (c) 24
- (d) 30





## Solutions

1 C

2 D

3 C

4 B

5 A

6 B

7 C

8 C

9 B

10 D

11 A

12 B

13 C

14 C

15 C

16 D

17 C

18 A

19 A

20 A

21 B

22 A

23 D

24 A

25 B

26 D

27 A

28 B

29 D

30 D