Fresno Math Circle **Grades 11-12** **Fall 2017**

October 18, 2017

Combinatorics

1. How many ways are there to put twelve identical balls into four boxes numbered 1, 2, 3, and 4?

2. How many ways are there to put twelve identical balls into four boxes numbered 1, 2, 3, and 4, so that each box contains at least one ball?

3. How many ways are there to put n identical balls into k boxes numbered 1, 2, ..., k?

4. How many ways are there to put n identical balls into k boxes numbered 1, 2, ..., k, so that each box contains at least one ball?

5. How many nonnegative integer solutions does the equation

$$x_1 + x_2 + x_3 + x_4 = 12$$

have?

6. How many positive integer solutions does the equation

$$x_1 + x_2 + x_3 + x_4 = 12$$

have?

7. How many nonnegative integer solutions does the inequality

$$x_1 + x_2 + x_3 \leq 12$$

have?

8. How many integer solutions does the system

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 12 \\ x_1 &\geq 3 \\ x_2 &\geq 2 \\ x_3 &\geq 1 \\ x_4 &\geq 0 \end{aligned}$$

have?

9. How many nonnegative integer solutions does the system

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 12 \\ x_1 &\leq 2 \\ x_2 &\geq 2 \\ x_3 &\geq 1 \\ x_4 &\geq 0 \end{aligned}$$

have?

10. How many nonnegative integer solutions does the system

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 12 \\ x_1 &\leq 1 \\ x_2 &\leq 2 \\ x_3 &\leq 3 \\ x_4 &\geq 4 \end{aligned}$$

have?

11. There are 7 boys and 4 girls in a math club. The photographer wants to seat them in a row so that no two girls are sitting next to each other. How many seating arrangements are possible?