MATH 150

Test 1

October 5, 2006

Name:

- No books or calculators are allowed.

- Please show all your work that you want to be considered in an organized fashion. Erase or cross out scratch work that you do not want to be considered.

- Please simplify your answers.

- Partial credit will be awarded for partially correct work or reasoning.

1. For the function $f(x)$ whose graph is given, state the value or that it does not exist (DNE).

$$\lim_{x \to 0^-} f(x) \qquad\qquad \lim_{x \to 0^+} f(x) \qquad\qquad \lim_{x \to 0} f(x) \qquad\qquad f(0)$$

$$\lim_{x \to 1^-} f(x) \qquad\qquad \lim_{x \to 1^+} f(x) \qquad\qquad \lim_{x \to 1} f(x) \qquad\qquad f(1)$$

$$\lim_{x \to 2^-} f(x) \qquad\qquad \lim_{x \to 2^+} f(x) \qquad\qquad \lim_{x \to 2} f(x) \qquad\qquad f(2)$$
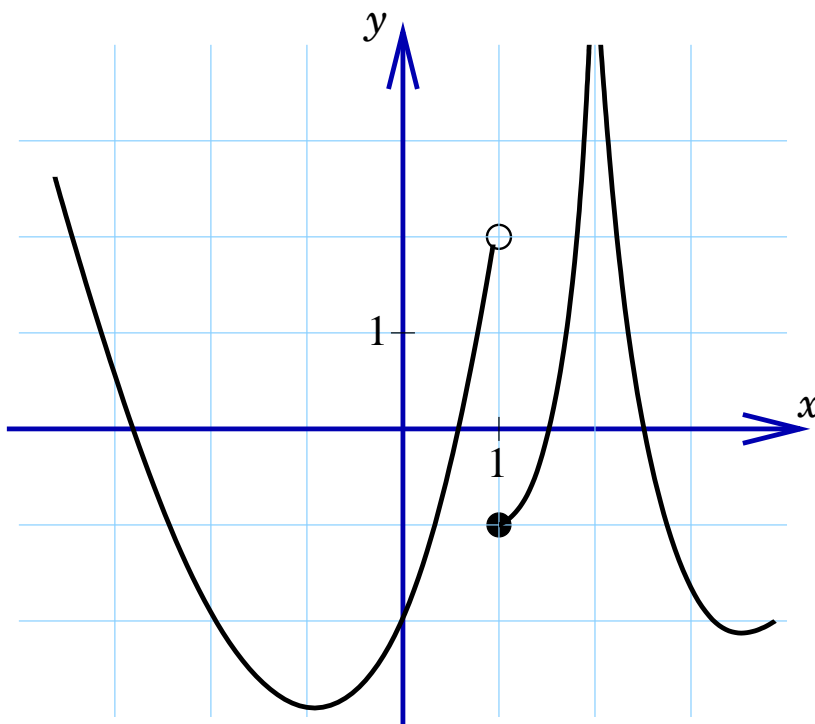
2. Analyze the continuity (or discontinuity) of the above graph at $x = 0$, $x = 1$, and $x = 2$. Explain in terms of limits, that is, by making clear reference to the definition of continuity.

3. Evaluate the following limits.

(a) $\lim\limits_{t \to 1} (t^3 - 2t^2 + 3t - 4)$

(b) $\lim\limits_{x \to 3} \dfrac{x^2 - 4x + 3}{x - 3}$

(c) $\lim\limits_{x \to \infty} \dfrac{4x^4 - x - 5}{3x^4 + 2}$

4. Find the derivative of each of the following functions.

(a) $f(x) = x^5 \sqrt{x}$

(b) $g(t) = \sin(t^2)$

(c) $y = \dfrac{x^2 + x - 4}{3x + 2}$

5. Use the definition of the derivative to find $f'(x)$ for the function $f(x) = x^2 - 3x$.

6. Find an equation of the tangent line to the curve $y = \cos(\pi x) - x^2$ at $x = 1$.

7. Find the exact x-values of the points at which the tangent lines to the graph of $f(x) = \dfrac{x^3}{3} - x^2 - 4x + 57$ are horizontal.

(Clarification: for example, $\sqrt{2}$ and π are exact values, while 1.414 and 3.14 are approximations.)

8. If $f(0) = 1$, $f'(0) = -2$, $g(0) = 3$, and $g'(0) = -4$, find the derivative of the product $f(x)g(x)$ at $x = 0$.

9. A ball is sent rolling down an inclined plane. The graph of its distance from the top is given below.
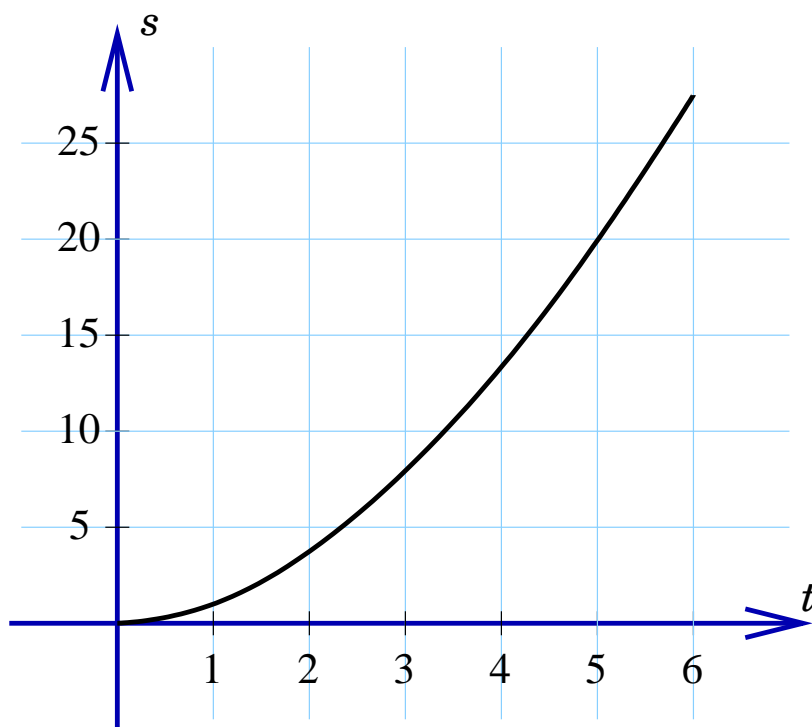
(a) Find the average velocity of the ball between 0 seconds and 5 seconds.

(b) Estimate the instantaneous velocity of the ball at 4 seconds. Explain why you think your estimate is correct.

10. Show that the equation $x^5 + 3x - 1 = 0$ has at least one real root in the interval $(-1, 1)$.

11. Find the value of c for which the function

$$f(x) = \begin{cases} 2x + 4 & \text{if } x \leq c \\ 7 - x & \text{if } x > c \end{cases}$$

is continuous everywhere.

For Extra Credit: Let $f(x) = \dfrac{x^2}{8}$. Note that $\lim\limits_{x \to 4} f(x) = 2$. Find a number δ such that $|f(x) - 2| < 0.5$ whenever $|x - 4| < \delta$. (You may sketch the graph of $f(x)$ and use it to determine δ.)
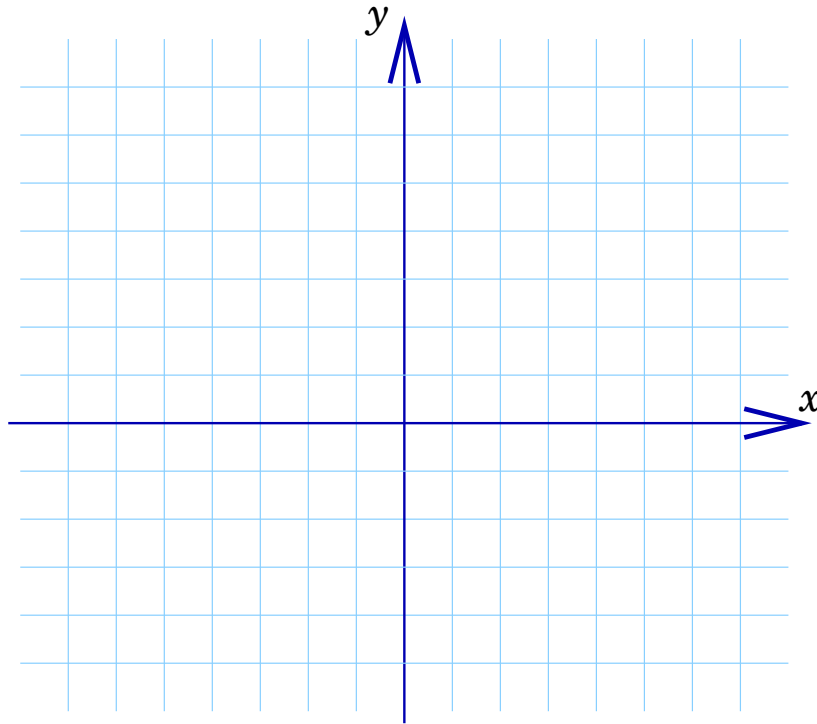
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Problem	Value	Score
1	12	
2	6	
3	9	
4	12	
5	10	
6	10	
7	10	
8	5	
9	10	
10	8	
11	8	
E.C.	5	
Total	100	

	Your scores so far	Out of
Pre-reading		20
WeBWorK		36
Quizzes		10
Labs		30
Test 1		100
Total		196

This page may be used as scratch paper