Comments re sample exams: no sample ever contains every example of every problem type possible. Use this to get an idea of the length and level of the exam. Use your study guide to be sure you are fully prepared!

Sample Exam 2, Calculus I, Fall 06

Directions: Show, in an organized fashion, any work you want considered. Erase or cross out scratch work that you do not want considered. (Partial credit will be awarded for partially correct work or reasoning.)

1. Consider the curve given by $x^3y^3 - 3xy^3 + 4y = 6$.

   (a) Use implicit differentiation to find $y'(x)$.

   (b) Verify that the point $(2, 1)$ lies on this curve.

   (c) What is the slope of the tangent line to this curve at $(2, 1)$?

2. For $f(t) = \dfrac{8}{t}$, find $f''(2)$ and $f'''(2)$.

3. Differentiate the following functions.

   (a) $f(x) = x \ln x$

   (b) $g(x) = \log_2(x^2 + x)$

   (c) $h(x) = 5 \sin^{-1}(x)$

   (d) $y = \arctan(2x^2)$

4. Use logarithmic differentiation to find $f'(x)$ for $f(x) = (\tan x)^{x^2}$.

5. A snowball is melting so that its radius is decreasing at a rate of 1 cm/min. Find the rate at which its volume is decreasing when the radius is 3 cm.

6. A boy starts walking west at 6 km/h from a point P. Five minutes later a girl starts walking north at 4 km/h from a point 15 km due south from P. At what rate is the distance between the kids changing 45 min after the girl starts walking? Is the distance increasing or decreasing at this instant?

7. Find the linearization of $g(x) = \sqrt{x}$ at $x = 1$ and use it to approximate $\sqrt{1.14}$.

8. Find the absolute maximum and minimum values of $f(x) = \sin x$ on the interval $\left[0, \frac{5\pi}{4}\right]$.

9. Show that the equation $\cos(x) + 2x + 1 = 0$ has exactly one real root.

10. Let $f(x) = x^4 + 4x^3 + 5$. Find the following.

(a) intervals of increase and decrease,

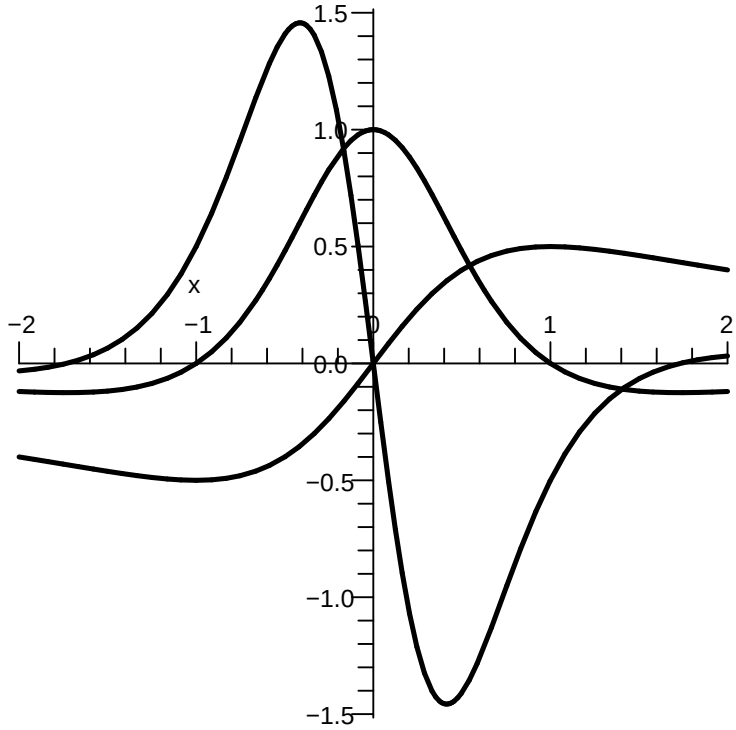
(b) local maximum and minimum points,

(c) intervals of concavity,

(d) inflection points.

(e) Sketch the graph of $y = f(x)$.

11. The figure below shows the graphs of f, f', and f''. Identify each curve, and explain your choices.



Extra Credit: Use implicit differentiation to verify the formula $\dfrac{d}{dx}(\arctan x) = \dfrac{1}{1+x^2}$.