Math 151

Solutions to selected homework problems

Section 1.1, Problem 8:

Let a, b, c be integers such that $a + b + c = 0$. Show that if n is an integer which is a divisor of two of the three integers, then it is also a divisor of the third.

Solution:

Let n be a divisor of two of the three integers a, b, c. Without loss of generality we can assume that $n|a$ and $n|b$. Therefore $a = nx$ and $b = ny$ for some $x, y \in \mathbb{Z}$. Then $c = -a - b = -nx - ny = n(-x - y)$. Since $-x - y \in \mathbb{Z}$, $n|c$.

Section 1.1, Problem 11:

Show that if $a > 0$, then $(ab, ac) = a(b, c)$.

Solution:

Let $d = (b, c)$. We will show that $ad = (ab, ac)$.

(i) Since $d|b$ and $d|c$, $b = dx$ and $c = dy$ for some $x, y \in \mathbb{Z}$. Then $ab = adx$ and $ac = ady$, thus $ad|ab$ and $ad|ac$.

(ii) Let $z|ab$ and $z|ac$. Then $ab = zk$ and $ac = zl$ for some $k, l \in \mathbb{Z}$.
Since $d = (b, c)$, $d = mb + nc$ for some $m, n \in \mathbb{Z}$.
Then $ad = amb + anc = abm + acn = zkm + zln = z(km_l n)$. Therefore $z|ad$.

(iii) We have $ad > 0$ because $a > 0$ and $d > 0$.