Math 151 Fall 2008

Practice problems for Test 2

Answers

1. (Note: feel free to show me your examples to make sure they are correct.)

group	order	abelian?	cyclic?
$\mathbb{Z}_5^*$	4	yes	yes
$\mathbb{Z}_6$	6	yes	yes
S_3	6	no	no
$\mathbb{Z}_4 \oplus \mathbb{Z}_2$	8	yes	no
$\mathbb{Z}$	∞	yes	yes
$GL_2(\mathbb{R})$	∞	no	no
$\{e\}$=trivial	1	yes	yes
D_5	10	no	no
$Mat_{2\times 3}(\mathbb{Z}_2)$	64	yes	no
$\mathbb{R}$	∞	yes	no

3. $\mathbb{R} \cong \mathbb{R}^+$; $\mathbb{Z}_2 \oplus \mathbb{Z}_8 \cong \mathbb{Z}_8 \oplus \mathbb{Z}_2$.

5. 4. $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \right\}$.

6. (a) 8

   (b) $H = \{0, 6, 12, 18\}$ is a cyclic subgroup. Generators: 6 and 18.
   $K = \{0, 4, 8, 12, 16, 20\}$ is a cyclic subgroup. Generators: 4 and 20.
   $H \cap K = \{0, 12\}$ is a cyclic subgroup. Generator: 12.
   $H \cup K = \{0, 4, 6, 8, 12, 16, 18, 20\}$ is not a subgroup.
   $H + K = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22\}$ is a cyclic subgroup. Generators: 2, 10, 14, 22.

7. (a) Yes. $\text{Ker}(f) = \{0\}$. Image $= 3\mathbb{Z}$. One-to-one. Not onto. Not isomorphism.

   (b) Yes. $\text{Ker}(f) = 4\mathbb{Z}$. Image $= \mathbb{Z}_4$. Not one-to-one. Onto. Not isomorphism.

   (c) Yes. $\text{Ker}(f) = 3\mathbb{Z}$. Image $= 2\mathbb{Z}_6$. Not one-to-one. Not onto. Not isomorphism.

   (d) No.

   (e) Yes. $\text{Ker}(f) = \{[0]\}$. Image $= \mathbb{Z}_{10}$. One-to-one. Onto. Isomorphism.

   (f) Yes. $\text{Ker}(f) = \{(x, -x)\}$. Image $= \mathbb{R}$. Not one-to-one. Onto. Not isomorphism.

   (g) Yes. $\text{Ker}(f) = \{(1, 1)\}$. Image $= \left\{ \begin{bmatrix} a & b \\ -2b & a+3b \end{bmatrix} \right\}$. One-to-one. Not onto. Not isomorphism.