

Practice problems for Test 3 - Answers

2. No. Yes. No.
3. Yes. Yes. No. No.
4. $q(x) = x^3 - 2x$, $r(x) = 7x + 1$
5. (a) $d(x) = x^3 + 4x^2 + 5x + 2$
 (b) $a(x) = 5$, $b(x) = 2x + 1$
6. $[x + 4]^{-1} = 3x^2 + 3x + 1$
7. -2 and -4
8. over \mathbb{Z} : $x^3 - 2$ is irreducible
 over \mathbb{Q} : still irreducible
 over \mathbb{R} : $(x - \sqrt[3]{2})(x^2 + \sqrt[3]{2}x + \sqrt[3]{4})$
 over \mathbb{C} : $(x - \sqrt[3]{2})\left(x + \frac{\sqrt[3]{2} + \sqrt[3]{2}\sqrt{3}i}{2}\right)\left(x + \frac{\sqrt[3]{2} - \sqrt[3]{2}\sqrt{3}i}{2}\right)$
 over \mathbb{Z}_3 : $(x + 1)^3$
9. $x^3 + x + 1$ and $x^3 + x^2 + 1$
10. use $p = 5$
11. (a) $(1, 1)$, $(1, 3)$, $(1, 5)$, $(1, 7)$, $(5, 1)$, $(5, 3)$, $(5, 5)$, $(5, 7)$
 (b) $(\pm 1, \pm 1, \pm 1)$ (8 elements total)
 (c) all elements of the form (a, b) where $a \neq 0$, $b \neq 0$