## Problem D (optional, for extra credit)

Last day to submit solutions: December 12, 2008

For which natural numbers $n, m$, and $k$, the function $f: \mathbb{Z}_{n} \rightarrow \mathbb{Z}_{m}$ given by $f\left([x]_{n}\right)=[k x]_{m}$

1. is well-defined?
2. preserves addition?
3. preserves multiplication?
4. is a group homomorphism?

5 . is a ring homomorphism?

