Math 151 **Fall 2008**

Project C (optional, for extra credit)

Last day to submit solutions: 29 October 2008

We have proved that if H is a subgroup of a finite group G, then the order of H divides the order of G. In this project we discuss whether or not the converse of this statement is true.

1. If $|G| = n$ and $m|n$, does G necessarily have a subgroup of order m?

   Hint 1: The answer is no. For example, you may show that 15 divides the order of S_5, but S_5 has no subgroup of order 15.

   Hint 2: It is a well-known fact that any group of order 15 is cyclic. This fact is helpful there, but if you choose to use it, then you have to prove it.

   Hint 3: To prove that any group of order 15 is cyclic, it may be easier to prove first that it is abelian, and then use this to conclude that it must be cyclic.

2. For each divisor m of the orfer of S_4, find a subgroup of S_4 of order m.