Math 151

Spring 2004

Test 2

Name: _____

1. (6 pts) Fill in the table. Provide brief explanations.

group	order	abelian?	cyclic?
$\mathbb{Z}_9^*$			
		yes	no
	24	no	

2. (5 pts) Are any of the groups $\mathbb{Z}_2 \oplus \mathbb{Z}_5$, $\mathbb{Z}_{10}$, $\mathbb{Z}_{10}^*$, D_5 isomorphic? Explain.

3. (4 pts) Find the order of $(2, 5)$ in $\mathbb{Z}_3 \oplus \mathbb{Z}_{10}$.

4. (4 pts) Show that the function $f : \mathbb{R} \to \mathbb{R}^*$ defined by $f(x) = 2^x$ is a homomorphism.

5. (5 pts) Find the kernel and the image of the homomorphism $f : \mathbb{Z}_{10} \to \mathbb{Z}_8$ defined by $f([x]_{10}) = [4x]_8$.

6. (6 pts) Let $G = GL_2(\mathbb{R})$ and $H = \{M \in G \mid \det(M) > 0\}$.

- Show that H is a subgroup of G.

- Show that H is normal in G.

Optional (for extra credit, 3 pts): Are $D_3 \times \mathbb{Z}_4$ and $D_4 \times \mathbb{Z}_3$ isomorphic?