Math 151

Spring 2004

Test 3

Name: _____

Each problem is worth 5 points.

1. Find the greatest common divisor of $f(x) = 6x^2 + 4x - 2$ and $g(x) = 3x^3 - 4x^2 - 2x + 1$ over R.

2. Is $x^3 - 2$ irreducible

   (a) over $\mathbb{Q}$?

   (b) over $\mathbb{Z}_5$?

   Explain.

3. Can a field have exactly

   (a) 1 element?

   (b) 2 elements?

   Explain.

4. Is the union of 2 ideals always an ideal? Prove or give a counterexample.

5. Find the multiplicative inverse of $[x]$ in $\mathbb{Z}_3 \; / < x^2 + x + 2 >$.

6. Let R and S be rings, and let $f : R \to S$ be an onto ring homomorphism. Prove that if R is commutative then so is S.

Optional (for extra credit, 3 pts): Is $\mathbb{Z}[x]/ < x^2 + 2 >$ a field?