Practice problems for Test 1

Answers

1. (a) 17

   (b) $m = 2$, $n = -1$

3. Solve the congruences:

   (a) $x \equiv 3 (\mathrm{mod}\ 8)$

   (b) No solutions

4. $x \equiv 156 (\mathrm{mod}\ 275)$

5. (a) It is the number of positive integers less than or equal to n that are relatively prime to n.

   (b) 8

6. 10; [901]

7. (a) Yes

   (b) No

9. (a) No. Transitive law is not satisfied, e.g. if $x = 1$, $y = 2$, $z = 3$.

   (b) Yes. Infinitely many equivalence classes containing 2 elements $\{x, -x\}$ for positive x, and one class containing 1 element $\{0\}$.

   (c) No. Reflexive law is not satisfied for $x = 0$.

   (d) Yes. 3 equivalence classes: $\mathbb{Z}_+$, $\{0\}$, and $\mathbb{Z}_-$.

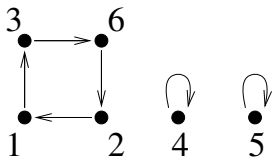
10. (a) $\sigma\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 1 & 6 & 2 \end{pmatrix}$, $\tau\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 6 & 2 & 3 & 4 \end{pmatrix}$.

    (b) No

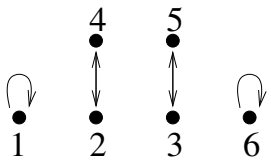
    (c) $\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 6 & 1 & 4 & 5 & 3 \end{pmatrix}$, $\tau^{-1} = \tau$.

    (d) $\sigma = (1362)$, $\tau = (24)(35)$

    (e)



σ



τ

    (f) $\sigma = (13)(36)(62)$

    (g) σ is odd; τ is even.