

## Practice problems for Test 2

## Answers

1. (Note: feel free to show me your examples to make sure they are correct.)

group	order	abelian?	cyclic?
$\mathbb{Z}_5^*$	4	yes	yes
$\mathbb{Z}_6$	6	yes	yes
$S_3$	6	no	no
$\mathbb{Z}_4 \oplus \mathbb{Z}_2$	8	yes	no
$\mathbb{Z}$	$\infty$	yes	yes
$GL_2(\mathbb{R})$	$\infty$	no	no
$\{e\}$ =trivial	1	yes	yes
$D_5$	10	no	no
$Mat_{2 \times 3}(\mathbb{Z}_2)$	64	yes	no
$\mathbb{R}$	$\infty$	yes	no

- 2.
- $\mathbb{R} \cong \mathbb{R}^+$
- ;
- $\mathbb{Z}_2 \oplus \mathbb{Z}_8 \cong \mathbb{Z}_8 \oplus \mathbb{Z}_2$
- .

4. 6.  $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} \right\}$ .

5. (a) 8

(b)  $H = \{0, 6, 12, 18\}$  is a cyclic subgroup. Generators: 6 and 18. $K = \{0, 4, 8, 12, 16, 20\}$  is a cyclic subgroup. Generators: 4 and 20. $H \cap K = \{0, 12\}$  is a cyclic subgroup. Generator: 12. $H \cup K = \{0, 4, 6, 8, 12, 16, 18, 20\}$  is not a subgroup. $H + K = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22\}$  is a cyclic subgroup. Generators: 2, 10, 14, 22.

6. (a) Yes.
- $\text{Ker}(f) = \{0\}$
- . Image =
- $3\mathbb{Z}$
- . One-to-one. Not onto.

(b) Yes.  $\text{Ker}(f) = 4\mathbb{Z}$ . Image =  $\mathbb{Z}_4$ . Not one-to-one. Onto.(c) Yes.  $\text{Ker}(f) = 3\mathbb{Z}$ . Image =  $2\mathbb{Z}_6$ . Not one-to-one. Not onto.

(d) No.

(e) Yes.  $\text{Ker}(f) = \{(x, -x)\}$ . Image =  $\mathbb{R}$ . Not one-to-one. Onto.

(f) No.

(g) Yes.  $\text{Ker}(f) = \{(1, 1)\}$ . Image =  $\left\{ \begin{bmatrix} a & b \\ -2b & a + 3b \end{bmatrix} \right\}$ . One-to-one. Not onto.

- 8 No. Yes. No.