Practice problems for Test 2

Hints

Note: These hints are intended for those who know the material but get stuck on one or two problems. If you don't know how to approach many of the practice problems, it means that you have to study all the material until you can do most of the problems on your own. It is certainly not sufficient to learn how to do these practice problems as the problems on the test will be different. The practice problems are only designed to show you the level of difficulty, and to identify weak areas.

1. As you read the book or review your notes, write out all examples of groups. For each group, determine its order (the number of elements in the group), whether it is abelian (the operation is commutative), and whether it is cyclic (see definition 3.2.5 on p.95).
2. If two groups are isomorphic, then they have the same properties. Thus if you can find a property that distinguishes the given groups (say, their have different order; or one group is cyclic and the other one is not; or one group has an element of a certain order, and the other one does not) then the groups are not isomorphic. To show that two groups are isomorphic, find an isomorphism, i.e. a function between the groups which preserves the operation, is 1-1, and is onto. Theorem 3.5.2 on p.121 may be used to show that 2 cyclic groups of the same order are isomorphic.
3. Use proposition 3.2.2 on p.93 to show that a subset is a subgoup. The order of an element is defined in definition 3.2.7 on p.97.
4. See definitions 3.2.7 on p.97 and 3.2.5 on p.95.
5. (a) See example 3.2.8 on p.97.

   (b) See definition 3.2.5 on p.95, definition 3.3.1 on p.105, exercise 12 on p.101.
6. Review 3.7
7. Just use the definition of a normal subgroup (definition 3.7.5 on p.140).
8. Again, use the definition of a normal subgroup. To show that a subgroup is not normal, you have to find a counterexample, i.e. find elements g in the group and h in the subgroup such that ghg^{-1} is not in the subgroup.