Practice problems for Test 3 - Answers

1. Yes; yes; yes; no; no

2. $q(x) = x^3 - 2x$, $r(x) = 7x + 1$

3.   (a) $d(x) = x^3 + 4x^2 + 5x + 2$

     (b) $a(x) = 5$, $b(x) = 2x + 1$

4. $[x + 4]^{-1} = 3x^2 + 3x + 1$

5. -2 and -4

6. over $\mathbb{Z}$: $x^3 - 2$ is irreducible

   over $\mathbb{Q}$: still irreducible

   over $\mathbb{R}$: $\left(x - \sqrt[3]{2}\right)\left(x^2 + \sqrt[3]{2}x + \sqrt[3]{4}\right)$

   over $\mathbb{C}$: $\left(x - \sqrt[3]{2}\right)\left(x + \dfrac{\sqrt[3]{2} + \sqrt[3]{2}\sqrt{3}i}{2}\right)\left(x + \dfrac{\sqrt[3]{2} - \sqrt[3]{2}\sqrt{3}i}{2}\right)$

   over $\mathbb{Z}_3$: $(x + 1)^3$

7. $x^3 + x + 1$ and $x^3 + x^2 + 1$

8. use $p = 5$

9.   (a) $R = \mathbb{Z}_6$, $e = [1]_6$, $S = 2\mathbb{Z}_6$, $e' = [4]_6$

10. $(1, 1)$, $(1, 3)$, $(1, 5)$, $(1, 7)$, $(5, 1)$, $(5, 3)$, $(5, 5)$, $(5, 7)$