Math 151 Spring 2004

Practice problems for Test 3 - Hints

1. See the chart with definitions, or the following definitions in the book:
   Com. ring: Def. 5.1.1 on p. 197.
   Ring: Def. 5.1.1 without commutativity for multiplication.
   Comm. ring with identity: Def. 5.1.1.
   Integral domain: Def. 5.1.6 on p. 203.
   Field: Def. 4.1.1 on p. 160.
   Also, see Theorem 5.3.6 on p. 221 (use the ring $\mathbb{Z}(i) = \{a + bi \mid a, b \in \mathbb{Z}\}$ and the ideal $I =< 2 >$).
2. See Example 4.2.1 and Theorem 4.2.1 on pp. 171-172.
3. (a) Use the Euclidean algorithm: Examples 4.2.3 and 4.2.4 on p. 176.
   (b) Follow the procedure for integers: Example 1.1.4 on p. 11.
4. See the proof of Prop. 4.4.5 on p. 190.
5. Use Prop. 4.3.1, and see Example 4.3.1 on p. 182.
6. Factor the polynomial over $\mathbb{R}$. Then use the quadratic formula.
   Over $\mathbb{Z}_3$: check each remainder modulo 3 (thus finding all the roots in $\mathbb{Z}_3$). If there is no root, the polynomial is irreducible. If there is a root, divide by $x-$root. Repeat.
   Another way: rewrite $x^3 - 2$ over $\mathbb{Z}_3$ so that it is either the sum or the difference of cubes, and factor.
7. There are $2^3 = 8$ polynomials of degree 3 over $\mathbb{Z}_2$. For each of them, find out whether or not it has roots (there are only two possible roots). A polynomial of degree 3 is irreducible iff it has no roots: Prop. 4.2.7.
8. See Theorem 4.3.6 on p. 184.
9. (a) See Example 5.1.3 on p. 201.
   (b) Since e is an identity in R, $ee' = e'$, $e \neq 0$.
   Since e' is an identity in S, $e'e' = e'$, $e' \neq 0$.
   In an integral domain, $ab = 0$ implies that either $a = 0$ or $b = 0$.
10. See the paragraph after Def. 5.2.8 on p. 215.
11. See Def. 5.3.1 on p.219.