**Math 151** **Spring 2004**

# Project C (optional, for extra credit)

Last day to submit solutions: 22 March 2004

We have proved that if $H$ is a subgroup of a finite group $G$, then the order of $H$ divides the order of $G$.

1. If $|G| = n$ and $m|n$, does $G$ necessarily have a subgroup of order $m$?
   **Hint 1:** The answer is no. Show that 15 divides the order of $S_5$, but $S_5$ has no subgroup of order 15.
   **Hint 2:** It is a well-known fact that any group of order 15 is cyclic. You may use this fact for part 1.
2. Prove that any group of order 15 is abelian.
3. Prove that any group of order 15 is cyclic.
4. For each divisor $m$ of the orfer of $S_4$, find a subgroup of $S_4$ of order $m$.