Math 151

Spring 2004

Test 3

Name: _________________________________

Each problem is worth 5 points.
1. Find the greatest common divisor of $f(x) = 6x^2 + 4x - 2$ and $g(x) = 3x^3 - 4x^2 - 2x + 1$ over $R$. 
2. Is $x^3 - 2$ irreducible
   (a) over $\mathbb{Q}$?
   (b) over $\mathbb{Z}_5$?
   Explain.
3. Can a field have exactly
   (a) 1 element?
   (b) 2 elements?
   Explain.
4. Is the union of 2 ideals always an ideal? Prove or give a counterexample.
5. Find the multiplicative inverse of \([x]\) in \(\mathbb{Z}_3 / \langle x^2 + x + 2 \rangle\).
6. Let $R$ and $S$ be rings, and let $f : R \rightarrow S$ be an onto ring homomorphism. Prove that if $R$ is commutative then so is $S$. 

Optional (for extra credit, 3 pts): Is $\mathbb{Z}[x]/<x^2 + 2>$ a field?