Math 151

Set
binary operation $+$
$\forall a, b, c \;\; a + (b + c) = (a + b) + c$
$\exists\, 0 \;\; \forall a \;\; a + 0 = 0 + a = a$
$\forall a \; \exists - a \;\; a + (-a) = (-a) + a = 0$

} group

$\forall a, b \;\; a + b = b + a$

} abel. group

binary operation $\cdot$
$\forall a, b, c \;\; a(bc) = (ab)c$
$\forall a, b, c \;\; a(b + c) = ab + ac, \;\; (a + b)c = ac + bc$

} ring

$\forall a, b \;\; ab = ba$

} comm. ring

$\exists 1 \; \forall a \;\; a1 = 1a = a$

} comm. ring with identity

$0 \neq 1$
$ab = 0 \Rightarrow a = 0$ or $b = 0$

} integ. domain

$\forall a \neq 0 \; \exists \; a^{-1} \;\; aa^{-1} = a^{-1}a = 1$

} field