Math 151

What types of questions will appear on the final

Give an example of something, e.g.

- Give an example of a nonabelian group of order 16.
- Give an example of a ring that is not an integral domain.
- Find an irreducible polynomial of degree 3 over $\mathbb{Z}_7$.

Yes/no questions, give brief explanations or examples, e.g.

- Can an abelian group have a nonabelian subgroup?
- Can a nonabelian group have an abelian subgroup?
- Is the function $\phi : \mathbb{Z}_{12} \to \mathbb{Z}_{16}$ given by $\phi([x]) = [4x + 5]$ a group homomorphism?
- Is a subring of a field necessarily a subfield?

Computational problems, e.g.

- How many positive divisors does 10! have?
- Solve the congruence $4x \equiv 2 \pmod{18}$
- Let $\sigma = (1253)$ and $\tau = (13)(24)$.
  1. Find $\sigma\tau$ and $\tau\sigma$.
  2. Find σ^{-1} and τ^{-1}.
  3. Find the orders of σ and τ.
  4. Are σ and τ cycles?
- Compute the kernel and the image of $\phi : \mathbb{Z}_{12} \to \mathbb{Z}_8$ given by $\phi([x]) = [4x]$.

Conceptual problems, e.g.

- Prove that the product of a cycle of length 3 and a cycle of length 4 cannot be equal to the product of two cycles of length 5.
- Prove that the identity mapping is the only ring homomorphism from $\mathbb{Z}$ to $\mathbb{Z}$.
- Let F be a field. Prove that there exist infinitely many irreducible polynomials over F. Hint: recall the proof that there are infinitely many prime numbers.