Math 151

Fall 2008

Test 1

Name: _____

- No books, notes, or calculators are allowed.
- Please show all your work. You can use the back of each page for scratch paper.
- Always explain your answers. Answers "yes" or "no" without explanations will receive 0 credit.

1. (4 pts) Solve the congruence $9x \equiv 2 \pmod{29}$.

2. (5 pts)

(a) List all the elements of $\mathbb{Z}_{15}^*$.

(b) Find the multiplicative inverse of $[7]$ in $\mathbb{Z}_{15}^*$.

3. (9 pts) Let $f : \mathbb{Z}_{15} \to \mathbb{Z}_3$ be given by $f([x]_{15}) = [2x]_3$.

(a) Show that f is a well-defined function.

(b) Is f one-to-one?

(c) Is f onto?

4. (8 pts) Consider the set of real numbers $\mathbb{R}$. For x and y in $\mathbb{R}$, let $x \sim y$ if $(x + y) \in \mathbb{Z}$.

   (a) Is $\sim$ reflexive?

   (b) Is $\sim$ symmetric?

   (c) Is $\sim$ transitive?

   (d) Is $\sim$ an equivalence relation?

5. (4 pts) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 1 & 5 & 6 & 3 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 1 \end{pmatrix}$.

(a) Find $\tau\sigma$.

(b) Write σ as a product of disjoint cycles.

Optional (for extra credit, 3 pts): Prove that the inverse of an even permutation is an even permutation, and that the inverse of an odd permutation is an odd permutation.