Math 151

Spring 2004

Test 1

1. (4 pts) Let a, b, $c \in \mathbb{Z}$, $c \neq 0$. Prove that $bc|ac \;\Leftrightarrow\; b|a$.

2. (5 pts) Solve the congruence $30x \equiv 18 (\mathrm{mod}\ 27)$.

3. (6 pts) Find

   (a) the multiplicative order

   (b) the multiplicative inverse

   of $[3]$ in $\mathbb{Z}_{11}^{*}$.

4. (4 pts) Is $f : \mathbb{Z}_{12} \to \mathbb{Z}_8$ given by $f([x]_{12}) = [3x]_8$ a well-defined function? Explain why or why not.

5. (5 pts) Consider the set of real numbers $\mathbb{R}$. For x and y in $\mathbb{R}$, let $x \sim y$ if $(x - y) \in \mathbb{Z}$. Show that $\sim$ is an equivalence relation, and describe the equivalence classes.

6. (6 pts) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 2 & 4 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 1 & 2 & 5 \end{pmatrix}$.

   (a) Find $\tau\sigma$.

   (b) Draw the associated diagram for σ.

   (c) Write σ as a product of disjoint cycles.

Optional (for extra credit, 3 pts): Does there exist an integer number m such that for any prime number p, $m \equiv p - 1 (\mathrm{mod}\ p)$? If such a number exists, find it. If not, prove that there is no such number.