Math 151

Fall 2008

Test 3

Name: ______________________________________

This test is due Wed, 12/10/08, at the beginning of the class period (9:15 am).

You may use your notes, the textbook for this class, and any materials posted on the course web page (such as homework solutions, practice test solutions, etc.). In fact, you are encouraged to do so whenever you are not sure about a notation or feel that you have to review a definition or examples, etc. There is no time limit. The goal is to learn the material well. You may use a calculator, but it probably won't be too helpful.

You may not use any other books, any materials other than the ones posted on the course web page, someone else's notes, etc. Please no joint work. Moreover, please do not even discuss the problems with anyone. (Limit all conversations to something of the sort "Have you finished the test yet?" – "Yes, I finished it last night".)

1. (6 points total) Consider the group $\mathbb{Z}_6 \times \mathbb{Z}_8$ and its subgroup $< (2, 4) >$. Find the order of each of the following:

   (a) (2 points) the group $\mathbb{Z}_6 \times \mathbb{Z}_8$,

   (b) (2 points) the subgroup $< (2, 4) >$,

   (c) (2 points) the factor group $(\mathbb{Z}_6 \times \mathbb{Z}_8)/ < (2, 4) >$.

2. (5 points) Find the greatest common divisor of $x^4 + x^3 + 2x^2 + x + 1$ and $x^3 + 2$ over $\mathbb{Z}_3$.

3. (4 points total) Let $f(x) = x^2 + 100x + n$.

(a) (2 points) Give an example of an integer n such that $f(x)$ is reducible over $\mathbb{Q}$. (Show that $f(x)$ is reducible for this value of n.)

(b) (2 points) Give an example of an integer n such that $f(x)$ is irreducible over $\mathbb{Q}$. (Prove that $f(x)$ is irreducible for this value of n.)

4. (9 points total) Recall that $\mathbb{R}$ is a group (under addition), a ring, and a field. Consider the subset $\mathbb{Z}[\sqrt{5}] = \{a + b\sqrt{5} \mid a, b \in \mathbb{Z}\}$ of $\mathbb{R}$.

   (a) (3 points) Is $\mathbb{Z}[\sqrt{5}]$ a subgroup of $\mathbb{R}$? Explain why or why not.

   (b) (3 points) Is $\mathbb{Z}[\sqrt{5}]$ a subring of $\mathbb{R}$? Explain why or why not.

   (c) (3 points) Is $\mathbb{Z}[\sqrt{5}]$ a subfield of $\mathbb{R}$? Explain why or why not.

5. (6 points) Let R and S be rings, and let $\phi : R \to S$ and $\theta : R \to S$ be ring homomorphisms. Show that $\{r \in R \mid \phi(r) = \theta(r)\}$ is a subring of R.

Optional (for extra credit, 3 points): Give an example of a non-commutative ring with exactly 10000 elements. (One point will be given for an example of a non-commutative ring of any finite order.)