Math 171 **Spring 2005**

Review sheet for test 1

The test will consist of 5 questions:

1. Give a definition.
   You should know the following definitions:
   (a) Absolute value (p. 8)
   (b) Least element of a set (p. 13)
   (c) Bounded above, upper bound, supremum of a set (p. 18)
   (d) Bounded below, lower bound, infimum of a set (p. 21)
   (e) One-to-one and onto function (p. 25)
   (f) Finite, countable, at most countable, uncountable set (p. 27)
   (g) Image and inverse image of a set under a function (p. 32)
   (h) Convergent sequence (p. 35)
   (i) Subsequence (p. 37)
   (j) Bounded above, bounded below, bounded sequence (p. 37)
   (k) Divergent to ∞ or $-\infty$ sequence (p. 41)
   (l) Increasing, strictly increasing, decreasing, strictly decreasing, monotone sequence (p. 45)
   (m) Nested sequence of intervals (p. 46)
   (n) Cauchy sequence (p. 49)
   (o) Limit of a function at a point (p. 58)
2. State a postulate or a theorem.
   You should know the following statements:
   (a) Field axioms (p. 2)
   (b) Order axioms (p. 4)
   (c) Well-ordering principle (p. 13)
   (d) Binomial formula (p. 16)
   (e) Completeness axiom (p. 19)
   (f) Archimedean principle (p. 19)
   (g) Density of rationals (p. 20)
   (h) Squeeze theorem for sequences (p. 39)
   (i) Comparison theorem (p. 43)
   (j) Monotone convergence theorem (p. 45)
   (k) Nested interval property (p. 46)
   (l) Bolzano-Weierstrass theorem (p. 47)
   (m) Cauchy theorem (p. 50)

(n) Sequential characterization of limits (p. 60)

3. Prove a theorem.
   You should know the statements and proofs of the following theorems:
   (a) If a set has an upper bound then it has infinitely many upper bounds; if a set has a supremum it has only one supremum. (p. 18)
   (b) Approximation property for suprema (p. 18)
   (c) DeMorgan's laws (p. 31)
   (d) A sequence can have at most one limit. (p. 36)
4. A homework problem or its slight modification. If you are using any theorems state clearly which ones.
   Review all required and recommended homework problems.
5. New problem (new in the sense that it has not been given in this class yet). If you are using any theorems state clearly which ones.
   Below are some practice problems:
   (a) Prove that for any natural number n, $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.
   (b) Use the binomial formula to prove that $1.01^{100} > 2.5$ (do not use a calculator).
   (c) Sketch the graph of $f(x) = |x| + |x - 2|$. (Hint: consider 3 cases: $x < 0$, $0 \leq x < 2$, and $x \geq 2$.) Is this function $f(x) : \mathbb{R} \to \mathbb{R}$ one-to-one? Is it onto?
   (d) Prove that the composition of two bijections is a bijection.
   (e) Find the limit of the sequence $x_n = \dfrac{n^2 + n\sin(n) + \cos(n^2)}{n^2 + 1}$ as $n \to \infty$.
   (f) Prove that the sequence $x_n = \sin(n) + \arctan(n)$ has a Cauchy subsequence.

   Below are some useful theorems. Although you will not be asked to state or prove any of them, you may use these theorems to solve problems.
   (a) Theorem 1.6 (p. 8)
   (b) Theorem 1.7 (p. 9)
   (c) Principle of mathematical induction (p. 13)
   (d) Lemma 1.14 (p. 16)
   (e) Theorem 1.28 (p. 22)
   (f) Theorem 1.29 (p. 22)
   (g) Theorem 1.31 (p. 25)
   (h) Theorem 2.8 (p. 37)
   (i) Theorem 2.11 (p. 40)
   (j) Theorem 2.12 (p. 40)
   (k) Theorem 2.15 (p. 41)
   (l) Theorem 3.8 (p. 61)
   (m) Theorem 3.10 (p. 63)