Math 171 **Spring 2005**

Review sheet for test 2

The test will consist of 5 questions:

1. Give a definition.
   You should know the following definitions:
   (a) Right-hand limit and left-hand limit of a function (p.66)
   (b) Limit at infinty (as $x \to \infty$ or $x \to -\infty$) (p. 67)
   (c) Infinite limit ($f(x) \to \infty$ or $f(x) \to -\infty$) (p. 67)
   (d) One-sided infinite limit (mentioned on p. 68)
   (e) Infinite limit at infinity (mentioned p. 68)
   (f) Continuous function (p. 72)
   (g) Composition of functions (p. 73)
   (h) Bounded function (p. 73)
   (i) Uniformly continuous function (p. 80)
   (j) Differentiable function (p. 85)
   (k) Continuously differentiable function (p. 89)
   (l) Increasing, strictly increasing, decreasing, strictly decreasing, monotone, strictly monotone function (p. 102)
2. State a theorem.
   You should know the following statements:
   (a) Sequential characterization of continuity (Theorem 3.21, p. 72)
   (b) Extreme value theorem (p. 74)
   (c) Intermediate value theorem (p. 75)
   (d) Characterization of uniform continuity on bounded open intervals (Theorem 3.40, p. 82)
   (e) The derivative of the sum, constant multiple, product, and quotient of functions (p. 92)
   (f) Chain rule (p. 92)
   (g) Rolle's theorem (p. 94)
   (h) L'Hospital's rule (p. 97)
   (i) Inverse function theorem (p. 103)
3. Prove a theorem.
   You should know the statements and proofs of the following theorems:
   (a) Sign-preserving property (p. 75)
   (b) A differentiable function is continuous. Not every continuous function is differentiable. (Theorem 4.4 and Example 4.5, p. 87-88)
   (c) Mean value theorem (Theorem 4.15 (ii), p. 96; either see the proof given in

class or state and prove part (i) first.)

(d) Increasing / decreasing test (Theorem 4.24, p. 102)

4. A homework problem or its slight modification. If you are using any theorems state clearly which ones.
Review all required and recommended homework problems.

5. New problem (new in the sense that it has not been given in this class yet). If you are using any theorems state clearly which ones.
Below are some practice problems:

(a) Let $f(x)$ be a continuous function on $\mathbb{R}$ such that $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$. Prove that $f(x)$ has a real root.

(b) Prove that for any $M > 0$, $f(x) = x^2$ is uniformly continuous on $[0, M]$.

(c) Give an example of a function continuous on $\mathbb{R}$ and differentiable at all points except -2 and 2.

(d) Give an example of a function $f(x)$ such that $\lim_{x\to 0^+} f(x) = -\infty$ and use the definition to show that your example satisfies this condition.

(e) Differentiate $f(x) = \left(1 + \frac{1}{x}\right)^x$.

(f) Evaluate the limit $\lim_{x\to 0}(\cos(3x))^{5/x}$

(g) For each of the following limits, find an example of a function $f(x)$ (and finite numbers a and L when needed) that satisfy the equation. For one-sided limits find examples such that $\lim_{x\to a} f(x)$ does not exist.

$\lim_{x\to a} f(x) = L$	$\lim_{x\to a^+} f(x) = L$	$\lim_{x\to a^-} f(x) = L$
$\lim_{x\to a} f(x) = \infty$	$\lim_{x\to a^+} f(x) = \infty$	$\lim_{x\to a^-} f(x) = \infty$
$\lim_{x\to a} f(x) = -\infty$	$\lim_{x\to a^+} f(x) = -\infty$	$\lim_{x\to a^-} f(x) = -\infty$
$\lim_{x\to \infty} f(x) = L$	$\lim_{x\to -\infty} f(x) = L$	
$\lim_{x\to \infty} f(x) = \infty$	$\lim_{x\to -\infty} f(x) = \infty$	
$\lim_{x\to \infty} f(x) = -\infty$	$\lim_{x\to -\infty} f(x) = -\infty$	

Suggestion: since the book skipped some definitions, write out a definition for each of the above limits. Make sure you understand them!

Below are some useful theorems. Although you will not be asked to state or prove any of them, you may use these theorems to solve problems.

(a) Theorem 3.14 (p. 67)

(b) Theorem 3.22 (p. 72)

(c) Theorem 3.24 (p. 73)

(d) Theorem 3.39 (p. 81)

(e) Remark 4.25 (p. 102)

(f) Theorem 4.26 (p. 103)