Math 171 **Spring 2005**

Review sheet for test 3

The test will consist of 5 questions:

1. Give a definition.
   You should know the following definitions:
   (a) Partition of an interval; norm of a partition; refinement of a partition (p. 107)
   (b) Upper Riemann sum; lower Riemann sum (p. 108)
   (c) Riemann integrable function (p. 110)
   (d) Upper integral; lower integral; integral (p. 112)
   (e) Riemann sum; convergent Riemann sums (p. 117)
   (f) Locally integrable function; improperly integrable function (p. 137)
   (g) Absolutely integrable function (p. 139) (Note: the definition in the book is incomplete. The function must also be locally integrable on (a, b).)
   (h) Conditionally integrable function (p. 139)
   (i) Infinite series; partial sum of a series (p. 154)
   (j) Convergent (divergent) series (p. 155)
   (k) Absolutely convergent series; conditionally convergent series (p. 165)
2. State a theorem.
   You should know the statements of the following theorems:
   (a) Linear property of Riemann integral (p. 119)
   (b) Comparison theorem for integrals (p. 121)
   (c) Fundamental theorem of Calculus (p. 127)
   (d) Change of variables (at least the case if f is continuous) (p. 130)
   (e) Convergence of a geometric series (p. 156)
   (f) Cauchy criterion (p. 157)
   (g) p-series test (p. 162)
   (h) Comparison test (p. 162)
   (i) Alternating series test (p. 175)
3. Prove a theorem.
   You should know the statements and proofs of the following theorems:
   (a) If $f : [a, b] \to \mathbb{R}$, then for any partitions P and Q of $[a, b]$, $L(f, P) \leq U(f, Q)$. (Remark 5.8, p. 110)
   (b) Integration by parts (p. 129)
   (c) Divergence of the harmonic series (Example 6.4, p. 155)
   (d) Divergence test (p. 156)

4. A homework problem or its slight modification. If you are using any theorems state clearly which ones. Review all required and recommended homework problems.
5. New problem (new in the sense that it has not been given in this class yet). If you are using any theorems state clearly which ones. Below are some practice problems:
   (a) Prove that if f is an even function then for any $a \in \mathbb{R}$, $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$, and if f is an odd function then for any $a \in \mathbb{R}$, $\int_{-a}^{a} f(x)dx = 0$.
   (b) Evaluate the limits (if they exist): $\int_0^{2\pi} |\sin x|dx$ $\quad$ $\int_0^{+\infty} |\sin x|dx$
   (c) Find all values of $p \in \mathbb{R}$ for which $f(x) = \dfrac{1}{x^p}$ is improperly integrable on $(1, +\infty)$.
   (d) Determine which of the following series converge (explain why or why not).
   $$\sum_{k=1}^{\infty} \cos\left(\frac{1}{k^2}\right) \qquad \sum_{k=1}^{\infty} \frac{k+3}{k^3+1} \qquad \sum_{k=1}^{\infty} \frac{k!}{2^k} \qquad \sum_{k=1}^{\infty} \frac{\cos(k\pi)}{\sqrt{k}}$$
   (e) Let $a_k \geq 0$, $b_k \geq 0$ for all $k \in \mathbb{N}$. Prove that if $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ converge then $\sum_{k=1}^{\infty} a_k b_k$ also converges.

   Below are some useful theorems and remarks. Although you will not be asked to state or prove any of them, you may use them to solve problems.
   (a) Remark 5.6 (p. 109)
   (b) Remark 5.7 (p. 109)
   (c) Theorem 5.10 (p. 111)
   (d) Theorem 5.15 (p. 113)
   (e) Theorem 5.16 (p. 114)
   (f) Theorem 5.20 (p. 119)
   (g) Corollary 5.23 (p. 122)
   (h) Theorem 5.42 (p. 138)
   (i) Theorem 5.43 (p. 138)
   (j) Theorem 6.10 (p. 157)
   (k) Theorem 6.12 (p. 160)
   (l) Theorem 6.16 (p. 163)
   (m) Theorem 6.23 (p. 167)
   (n) Theorem 6.24 (p. 167)