Math 171 **Optional assignment** **Spring 2005**

due Wednesday, April 13

This assignment is only for those who did not attend the talk on Friday, April 4.

Do any 4 of the following problems. Note: you do not have to use my hints. There are many possible solutions of these problems. Find any solutions you like. You may use books, internet, anything you want... You can get up to 10 points for this assignment.

1. Prove that the product of any four consecutive numbers is divisible by 24 (hint: a number is divisible by 24 if and only if it is divisible by both 3 and 8).
2. Find a proof of (1) that does not use divisibility properties (hint: use a combinatorial argument for $\binom{n}{4}$).
3. Prove that for any complex numbers z_1 and z_2, $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$.
4. Find a proof of (3) that does not use trigonometry (hint: use geometry of the complex plane).
5. Prove that for any real number t, $e^{it} = \cos t + i \sin t$ (hint: use power series).
6. Find a proof of (5) that does not use power series (warning: this is a tricky problem).
7. Prove that for any positive real numbers $a_1, \ldots, a_n$,
$$\frac{a_1 + a_2 + \ldots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \ldots a_n}.$$
8. Consider the following experiment. We choose a random real number p in the interval $[0, 1]$. Then we build an unfair coin such that when we flip it the probability of getting "heads" is equal to p. Then we flip the coin 2000 times.
For this experiment, what is the probability that we get "heads" exactly 1000 times? (warning: this is a tricky problem.)