MATH 171

Test 1

February 28, 2005

Name:

- No books, notes, or calculators are allowed.

- Please show all your work.

1. (6 points) Give the definition of a Cauchy sequence.

2. (6 points) State the Well-ordering Principle.

3. (14 points) State and prove the Approximation Property for Suprema.

4. (12 points) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = (x+1)^2 - 3$ and let $E = (-3, 0]$. Find $f(E)$ and $f^{-1}(E)$. (Explain how you find these!)

5. (12 points) Prove that for all $n \in \mathbb{N}$,

$$1 + 2 + 3 + \ldots + (n-2) + (n-1) + n + (n-1) + (n-2) + \ldots + 3 + 2 + 1 = n^2.$$

6. (**For extra credit**, 10 points) Prove or disprove each of the following statements:

(a) If $\lim_{x \to a} f(x) = L$ then $\lim_{x \to a} |f(x)| = |L|$.

(b) If $\lim_{x \to a} |f(x)| = |L|$ then $\lim_{x \to a} f(x) = L$ or $\lim_{x \to a} f(x) = -L$.