MATH 171

Test 1

February 28, 2005

Name:______________________________

- No books, notes, or calculators are allowed.
- Please show all your work.
1. (6 points) Give the definition of a Cauchy sequence.

2. (6 points) State the Well-ordering Principle.
3. (14 points) State and prove the Approximation Property for Suprema.
4. (12 points) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = (x + 1)^2 - 3$ and let $E = (-3, 0]$. Find $f(E)$ and $f^{-1}(E)$. (Explain how you find these!)
5. (12 points) Prove that for all \( n \in \mathbb{N} \),

\[
1 + 2 + 3 + \ldots + (n - 2) + (n - 1) + n + (n - 1) + (n - 2) + \ldots + 3 + 2 + 1 = n^2.
\]
6. **(For extra credit, 10 points)** Prove or disprove each of the following statements:

(a) If \( \lim_{x \to a} f(x) = L \) then \( \lim_{x \to a} |f(x)| = |L| \).

(b) If \( \lim_{x \to a} |f(x)| = |L| \) then \( \lim_{x \to a} f(x) = L \) or \( \lim_{x \to a} f(x) = -L \).