MATH 171

Test 2

April 8, 2005

Name:______________________________

• No books, notes, or calculators are allowed.

• Please show all your work.
1. (6 points) Give the definition of $\lim_{x \to \infty} f(x) = L$.

2. (6 points) State Rolle’s theorem.
3. (14 points) State and prove the sign-preserving property.
4. (12 points) Find all $a \in \mathbb{R}$ such that $f(x) = \frac{ax + 2}{x + 1}$ is strictly increasing on $(1, 2)$. 
5. (12 points) Let $f(x)$ and $g(x)$ be uniformly continuous on $\mathbb{R}$. Prove that $(f + g)(x)$ is uniformly continuous on $\mathbb{R}$.
6. (For extra credit, 10 points) Prove or disprove each of the following statements:

(a) If a function is continuously differentiable on \( \mathbb{R} \) then it is twice differentiable on \( \mathbb{R} \).

(b) If a function is continuously differentiable 100 times on \( \mathbb{R} \) then it is differentiable 101 times on \( \mathbb{R} \).