Math 250 **Maria Nogin**

Practice test 1 - Answers

1. No
2. $(x-4)^2+(y+3)^2+(z-2)^2=9$. The sphere and the yz-plane do not intersect.
3. (a) $< -2, 2, 4 >$
   (b) 0
   (c) $< 2, -10, 6 >$
   (d) $\frac{\pi}{2}$
4. $\sqrt{3}$
5. (a) $x-1=y-4=\frac{z+2}{5}$
   (b) $\frac{x-1}{2}=\frac{y-4}{5}=\frac{z+2}{3}$
   (c) same as in (b)
6. (a) $23x-8y-3z+3=0$
   (b) $2x+5y+3z-16=0$
   (c) same as in (b)
   (d) $2x-y-z=0$
7. (a) Lines do not intersect.
   (b) Point $(-5,-4,-6)$.
   (c) Point $(-3,-2,-4)$.
   (d) Line $\frac{3x+1}{-22}=\frac{3y+2}{7}=z$.
   (e) Ellipse (in the xz-plane) given by $x^2+\frac{z^2}{4}=1$, $y=0$.
8. Position the coordinate system so that the shell is along the z-axis and its center is at the origin. Then the shell is described by:
   $36 \le x^2+y^2 \le 49$, $-10 \le z \le 10$ in rectangular coordinates, or
   $6 \le r \le 7$, $-10 \le z \le 10$ in cylindrical coordinates.
9. (a) Plane
   (b) Circlular cylinder
   (c) Sphere

(d) Half-plane

(e) Half-cone

10. Let $r(t) =< \cos t, t^2, t^4 >$.

    (a) $< -\sin t, 2t, 4t^3 >$

    (b) $< -2, \frac{\pi^3}{3}, \frac{\pi^5}{5} >$

    (c) No

    (d) $\int_0^1 \sqrt{\sin^2 t + 4t^2 + 16t^6}\ dt$

11. $v(\pi) =< 0, 3, -2 >$, $a(\pi) =< 2, 0, 0 >$, speed is $\sqrt{13}$.

12. (a) If the particle meet at time t, then $2\cos t = t - 5$, $3 = t$, and $\pi - t = 2t - 6$. This system is inconsistent because the second equation gives $t = 3$, but this value does not satisfy the third equation ($\pi - 3 \neq 0$).

    (b) At $t = \pi$, the first particle is at $< 2\cos\pi, 3, \pi - \pi >=< -2, 3, 0 >$. At $t = 3$, the second particle is at $< 3 - 5, 3, 2 \cdot 3 - 6 >=< -2, 3, 0 >$. Since $3 < \pi$, the second particle passes throught this point earlier.

13. $\{(x, y, z) \mid x+2y-1 \geq 0,\ z > 0\}$. This is the set of points above the half of the xy-plane given by $y \geq -x/2 + 1/2$.
    In other words, the xy-plane and the vertical plane given by $x + 2y - 1 = 0$ divide the space into 4 parts. The domain of the given function is one of these four parts (the one above the xy-plane and containing points with large positive x- and y-coordinates).

14. The level curves are given by $y - e^x = k$, or $y = e^x + k$. Draw a few of these curves, say, for $k = 0, 1, -1, 2, -2$. For $k = 0$ we have $y = e^x$. For $k = 1$ we have $y = e^x + 1$ which is obtained by shifting $y = e^x$ 1 unit upward. For $k = -1$ we have $y = e^x - 1$ which is obtained by shifting $y = e^x - 1$ 1 unit downward. And so on.

15. The level surfaces are given by $x^2 + 3y^2 + 5z^2 = k$, or $\frac{x^2}{k} + \frac{y^2}{k/3} + \frac{z^2}{k/5} = 1$ and are ellipsoids.