Math 250 **Maria Nogin**

Practice test 1

Test 1 is on Monday, October 2, from 10:00 - 10:50 AM. The actual test will consist of 5 problems. (We will go over the test from 10:50 - 11:15.)

1. Determine whether the points $A(1,0,0)$, $B(0,1,1)$, $C(-2,2,2)$ lie on a straight line.
2. Find an equation of the sphere with center $(4,-3,2)$ and radius 3. What is the intersection of this sphere with the yz-plane?
3. Let $a =< 1,2,3 >$ and $b =< -3,0,1 >$. Find the following:
   (a) $a + b$
   (b) $a \cdot b$
   (c) $a \times b$
   (d) the angle between a and b
4. Find the area of the triangle whose vertices are $P(1,2,0)$, $Q(0,1,2)$, and $R(1,0,2)$.
5. Find equations of the line
   (a) through $(1,4,-2)$ and $(2,5,3)$
   (b) through $(1,4,-2)$ and parallel to $< 2,5,3 >$
   (c) through $(1,4,-2)$ and orthogonal to $2x + 5y + 3z + 4 = 0$
6. Find an equation of the plane
   (a) through $(1,4,-2)$, $(2,5,3)$, and $(0,0,1)$
   (b) through $(1,4,-2)$ and parallel to $2x + 5y + 3z + 4 = 0$
   (c) through $(1,4,-2)$ and orthogonal to $< 2,5,3 >$
   (d) through $(1,4,-2)$ and containing the line $x = y - 1 = z + 1$
7. Describe the intersections of the following, and find their coordinates or parametric equations:
   (a) the lines $x = y - 1 = z + 1$ and $x - 2 = 2y + 1 = \frac{z}{2}$
   (b) the lines $x = y - 1 = z + 1$ and $x - 2 = 2y + 1 = \frac{z-8}{2}$
   (c) the line $x = y - 1 = z + 1$ and the plane $x + 4y - 2z + 3 = 0$
   (d) the planes $x + 4y - 2z + 3 = 0$ and $2x + 5y + 3z + 4 = 0$
   (e) the ellipsoid $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$ and xz-plane

8. (Section 12.7, problem 63) A cylindrical shell (see figure 2 on page 455) is 20 cm long, with inner radius 6 cm and outer radius 7 cm. Write inequalities that describe the shell in an appropriate coordinate system. Explain how you have positioned the coordinate system with respect to the shell.

9. Identify the surface:

   (a) $x = 3$

   (b) $r = 3$

   (c) $\rho = 3$

   (d) $\theta = \frac{\pi}{4}$

   (e) $\phi = \frac{\pi}{4}$

10. Let $r(t) =< \cos t, t^2, t^4 >$.

    (a) Find $r'(t)$.

    (b) Find $\int_0^{\pi} r(t)dt$.

    (c) Is the curve given by $r(t)$ smooth?

    (d) Set up (but do not evaluate) an integral for the length of this curve from $t = 0$ to $t = 1$.

11. Find the velocity, acceleration, and speed of a particle with position function $r(t) =< 2\cos t, 3t, 2\sin t >$ at $t = \pi$.

12. One particle has position function $r(t) =< 2\cos t, 3, \pi - t >$ and another one has position function $r(t) =< t - 5, t, 2t - 6 >$.

    (a) Show that these particles never meet.

    (b) Show that both particles pass through the point $(-2, 3, 0)$. Which particle passes through this point first?

13. Find and describe the domain of $f(x, y, z) = \sqrt{x + 2y - 1} + \ln(z)$.

14. Draw a countour map of the function $f(x, y) = y - e^x$ showing several level curves.

15. Describe the level surfaces of the function $f(x, y, z) = x^2 + 3y^2 + 5z^2$.