Math 250 **Maria Nogin**

Practice test 2

Test 2 is on Monday, November 6, from 10:00 - 10:50 AM. This test is on sections 14.2-15.6. The actual test will consist of 5 problems (and one problem for extra credit). We will go over the test from 10:50 - 11:15.

1. Show that the limit $\lim_{(x,y)\to(0,0)} \frac{x(x-y)}{x^2+y^2}$ does not exist.

2. Let $f(x, y) = \sin x + \cos y + e^{xy}$. Find the first and second partial derivatives of f.

3. Let $f(x, y) = \sqrt{x + y^2}$.
   (a) Find an equation of the tangent plane to the surface given by $z = f(x, y)$ at $(8, 1, 3)$.
   (b) Find the linearization of $f(x, y)$ at $(8, 1)$.

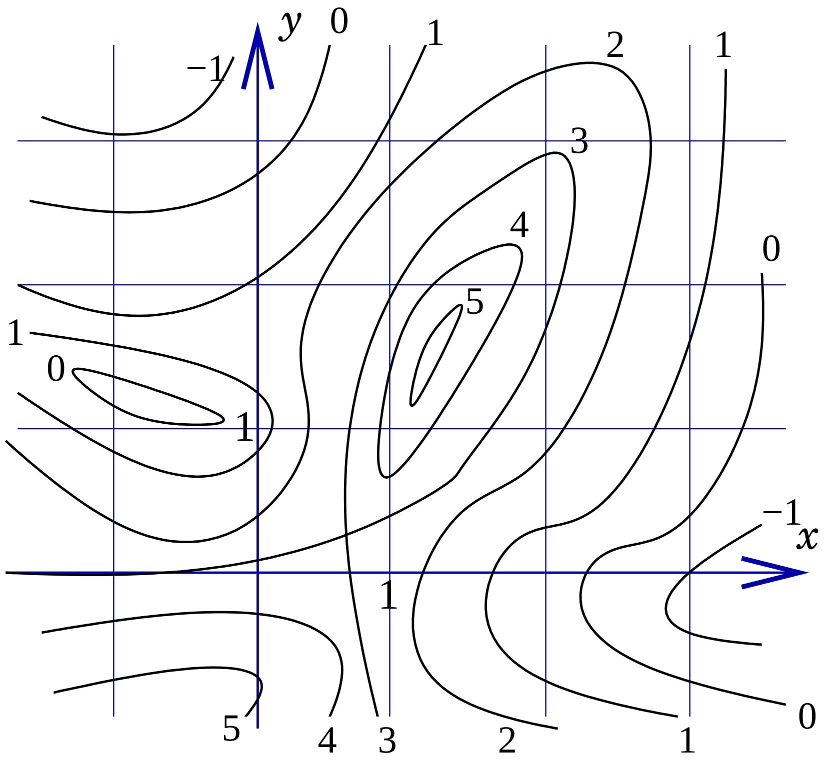
4. Use the chain rule to find the indicated partial derivative(s) for the given function.
   (a) $z = x \ln(x + y)$, $x = t^2$, $y = t^3$; find $\frac{dz}{dt}$
   (b) $z = \arctan x + y^3$, $x = t^4 + 2s$, $y = -st$; find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$
   (c) $u = x^2 + y^2 - z^2$, $x = 5t + 1$, $y = e^t$, $z = \sin t$; $\frac{du}{dt}$ at $t = 0$

5. Let $f(x, y) = x\sqrt{y}$. Find the following:
   (a) $\nabla f(x, y)$,
   (b) $\nabla f(3, 4)$,
   (c) the directional derivative of f at $(3, 4)$ in the direction of the vector $v = < 1, -1 >$,
   (d) the maximum rate of change of f at $(3, 4)$ and the direction in which it occurs.

6. Let $f(x, y) = x^3 + y^3 - 3(x + y)$. Find the following:
   (a) the critical points of f,
   (b) the local maximum and minimum values of f,
   (c) the absolute maximum and minimum values of f on $\mathbb{R}^2$,
   (d) the absolute maximum and minimum values of f on the circle given by $x^2 + y^2 = 1$,
   (e) the absolute maximum and minimum values of f on the disk $\{(x, y) \mid x^2 + y^2 \leq 1\}$.

7. A contour map is shown for a function f. Estimate the following:
   (a) the value of $\iint_R f(x, y)dA$, where $R = [-1, 3] \times [0, 3]$,

(b) the average value of of f on R.



8. Evaluate the following integrals:

   (a) $\int_0^1 \int_0^2 3dxdy$

   (b) $\int_0^6 \int_0^2 (x^2 + xy + 1)dydx$

   (c) $\int_0^1 \int_{-y}^{y} (x^2 + xy + 1)dxdy$

   (d) $\iint_R \sin(x^2 + y^2)dA$, where $R = \{(x, y) \mid x^2 + y^2 \leq 9,\ x \geq 0,\ y \geq 0\}$ (hint: change to polar coordinates)

9. Find the mass and center of mass of the lamina that occupies the region

$$D = \left\{(x, y) \mid 0 \leq y \leq \cos x, 0 \leq x \leq \frac{\pi}{2}\right\}$$

   and has the density function $\rho(x, y) = x$.

10. Find the area of the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the xy-plane.