Math 250 **Maria Nogin**

Practice final - answers

1. (a) plane containing the x-axis and the line $y = z$ in the yz-plane
   (b) vertical cylinder with radius 2
   (c) solid containing all points outside the sphere with radius 3 and center at the origin
   (d) vertical half-plane containing the negative x-axis
   (e) paraboloid with vertex at $(0, 0, 0)$, opening downward
   (f) plane through $(1, 0, 2)$ and parallel to vectors $< 2, -1, 4 >$ and $< 0, 3, 5 >$ (section 16.6: will not be on the final)

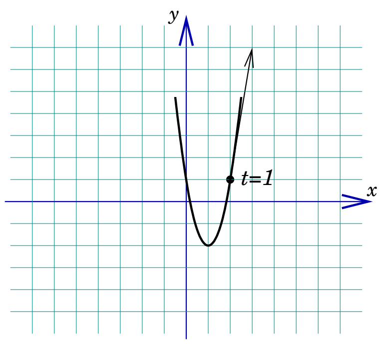
2. $\left\langle \frac{3\sqrt{2}}{\sqrt{7}}, \frac{6\sqrt{2}}{\sqrt{7}}, \frac{9\sqrt{2}}{\sqrt{7}} \right\rangle$

3. (a) -4
   (b) $< 8, -6, -3 >$
   (c) $\arccos\left(-\frac{4}{5\sqrt{5}}\right)$
   (d) $\sqrt{109}$

4. $x = 1 + 2t,\ y = -4t,\ z = -3 + 5t;\quad \frac{x-1}{2} = -\frac{y}{4} = \frac{z+3}{5}$

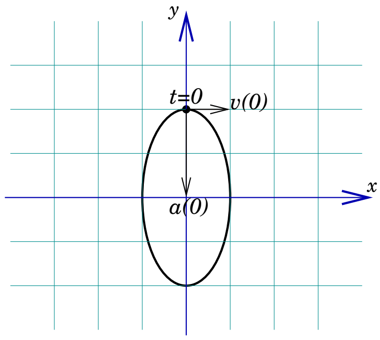
5. $9x - 12y - 11z = 0$

6. $r'(t) =< 1, 6t >,\ \ r'(1) =< 1, 6 >$

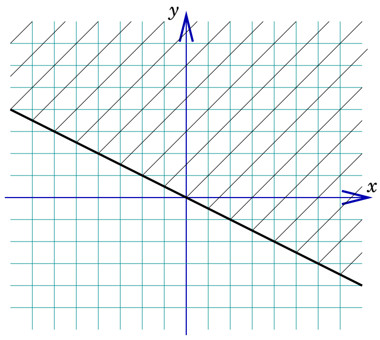


7. $\frac{13\sqrt{13} - 8}{27}$

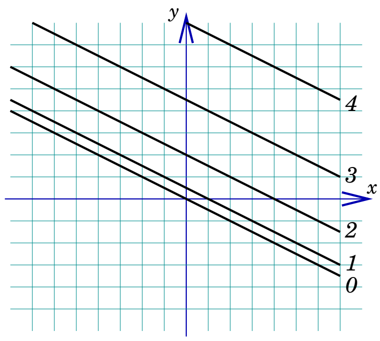
8. $v(t) =< \cos t, -2\sin t >;\ \ a(t) =< -\sin t, -2\cos t >;\ \ |v(t)| = \cos^2 t + 4\sin^2 t;$
   $v(0) =< 1, 0 >;\ \ a(0) =< 0, -2 >$



9. (a) $\text{Domain}(f) = \{(x, y) \mid x + 2y \geq 0\}$



(b) Level curves are lines:



(c) $f_x = \dfrac{1}{2\sqrt{x+2y}}$; $f_y = \dfrac{1}{\sqrt{x+2y}}$;

$f_{xx} = -\dfrac{1}{4(x+2y)^{3/2}}$; $f_{yy} = -\dfrac{1}{(x+2y)^{3/2}}$; $f_{xy} = -\dfrac{1}{2(x+2y)^{3/2}}$;

(d) $\left\langle \dfrac{1}{2\sqrt{x+2y}}, \dfrac{1}{\sqrt{x+2y}} \right\rangle$

(e) $\dfrac{1}{6\sqrt{2}}$

(f) $L(x, y) = \frac{1}{6}(x-1) + \frac{1}{3}(y-4) + 3$

(g) maximum value: $\sqrt{3+\sqrt{5}}$; minimum value: $\sqrt{3-\sqrt{5}}$

10. $\text{Domain}(f) = \mathbb{R}^3$; level surfaces are paraboloids $z = x^2 + y^2 - k$, $k \in \mathbb{R}$

11. If $(x, y) \to (0,0)$ along the x-axis, then $y = 0$, so $\lim_{(x,y)\to(0,0)} \dfrac{6x^3y}{2x^4+y^4} = 0$; if $(x, y) \to (0,0)$ along the line $y = x$, then $\lim_{(x,y)\to(0,0)} \dfrac{6x^3y}{2x^4+y^4} = \lim_{(x,x)\to(0,0)} \dfrac{6x^4}{3x^4} = 2$; since these two limits are not equal, the limit $\lim_{(x,y)\to(0,0)} \dfrac{6x^3y}{2x^4+y^4}$ does not exist.

12. (a) 40

    (b) $\dfrac{5}{24}$

    (c) $-\dfrac{14}{3}$

    (d) $\dfrac{\sqrt{34}(16e^4 - e)}{9}$

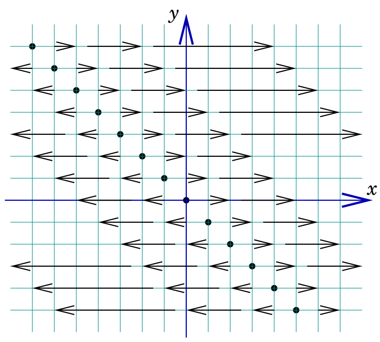
    (e) $\dfrac{36\sqrt{3} + 16\sqrt{2} - 8}{105}$

13. 4π

14. 128π

15. $8uv$

16. All vectors are horizontal, the magnitute and direction of each vector are given by $x + y$ (where (x, y) is the starting point of the vector).



17. (a) $\dfrac{\partial Q}{\partial x} = e^4$, $\dfrac{\partial P}{\partial y} = e^4$; $f = xe^y$

    (b) $\text{curl}(F) = 0$; $f = x^2y + y^2z$

18. 63.6

19. curl $F = (xz^2 - xy^2)\mathbf{i} + (x^2y - yz^2)\mathbf{j} + (y^z - x^2z)\mathbf{k}$; div $F = 6xyz$

20. $4e - 1$ (section 16.8: will not be on the final)