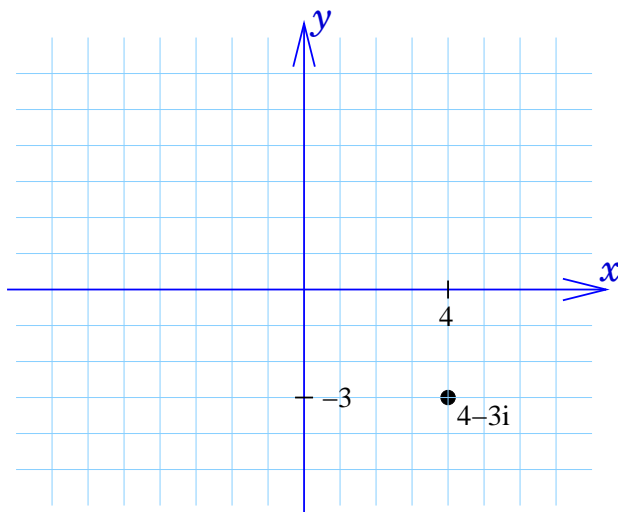


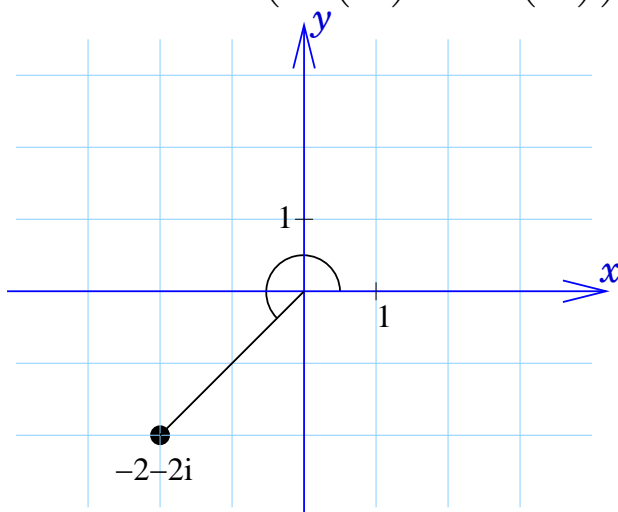
MATH 5

Practice Test 3 – Answers

- $5 - i$
- $-5i$
- $|4 - 3i| = \sqrt{4^2 + 3^2} = 5$



4. $r = 2\sqrt{2}$, $\phi = \frac{5\pi}{4}$, $-2 - 2i = 2\sqrt{2} \left(\cos \left(\frac{5\pi}{4} \right) + i \sin \left(\frac{5\pi}{4} \right) \right)$



5. Using a product-to-sum identity:
- $$\cos(75^\circ) \sin(105^\circ) = \frac{1}{2} (\sin(180^\circ) + \sin(30^\circ)) = \frac{1}{2} \left(0 + \frac{1}{2} \right) = \frac{1}{4}$$

$$\begin{aligned}
6. \quad & \sin(5x) - \sin(x) = 0 \\
& 2 \cos(3x) \sin(2x) = 0 \\
& \cos(3x) = 0 \quad \text{or} \quad \sin(2x) = 0 \\
& 3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2} \quad \text{or} \quad 2x = 0, \pi, 2\pi, 3\pi \\
& x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \quad \text{or} \quad x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \\
& \text{Answer: } 0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}
\end{aligned}$$

$$7. \text{ Let } d \text{ be the distance. Then } \tan 10^\circ = \frac{50}{d}, \text{ so } d = \frac{50}{\tan 10^\circ}.$$

$$\begin{aligned}
8. \quad & c^2 = 49 + 16 - 56 \cos\left(\frac{\pi}{4}\right) = 65 - 28\sqrt{2} \\
& c = \sqrt{65 - 28\sqrt{2}} \\
& \cos \alpha = \frac{16 + 65 - 28\sqrt{2} - 49}{8\sqrt{65 - 28\sqrt{2}}} = \frac{8 - 7\sqrt{2}}{2\sqrt{65 - 28\sqrt{2}}} \\
& \alpha = \arccos\left(\frac{8 - 7\sqrt{2}}{2\sqrt{65 - 28\sqrt{2}}}\right) \\
& \cos \beta = \frac{49 + 65 - 28\sqrt{2} - 16}{14\sqrt{65 - 28\sqrt{2}}} = \frac{7 - 2\sqrt{2}}{\sqrt{65 - 28\sqrt{2}}} \\
& \beta = \arccos\left(\frac{7 - 2\sqrt{2}}{\sqrt{65 - 28\sqrt{2}}}\right)
\end{aligned}$$

$$\begin{aligned}
9. \quad & \angle \gamma = 45^\circ \\
& AC = \frac{AB \sin 75^\circ}{\sin 45^\circ} = \frac{6 \cdot \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 3 \cdot (\sqrt{3} + 1) \\
& BC = \frac{AB \sin 60^\circ}{\sin 45^\circ} = 3\sqrt{6}
\end{aligned}$$

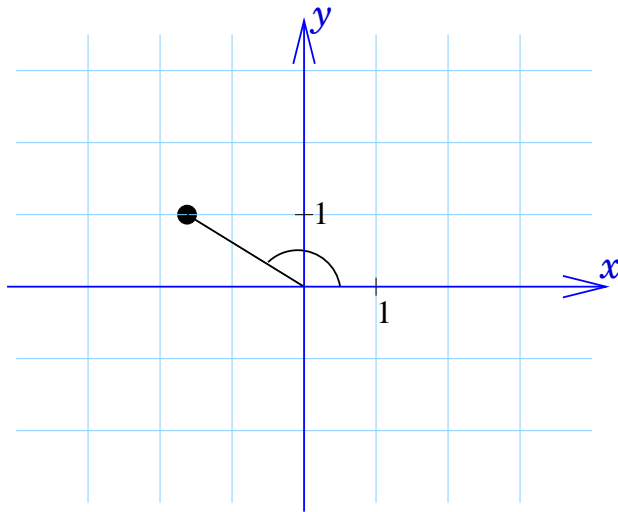
$$10. \text{ Area} = \frac{1}{2} AB \cdot AC \sin(60^\circ) = \frac{1}{2} \cdot 6 \cdot 3 \cdot (\sqrt{3} + 1) \cdot \frac{\sqrt{3}}{2} = \frac{9(3 + \sqrt{3})}{2}$$

$$11. \text{ Area} = \frac{1}{2} \cdot \frac{3\pi}{4} \cdot 12^2 = 54\pi$$

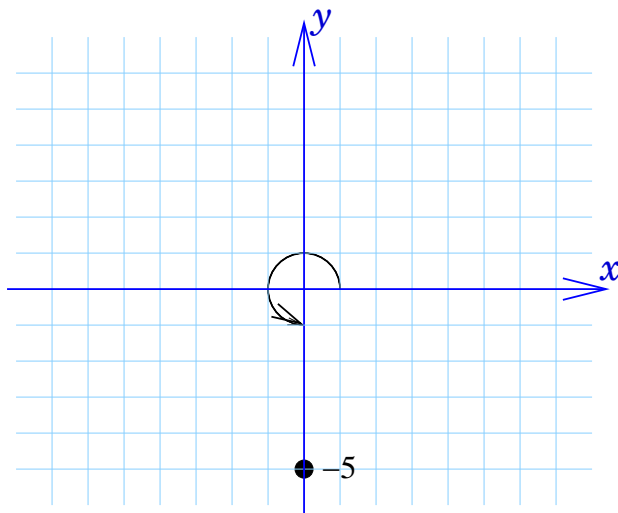
$$12. \frac{h}{r} = \tan(60^\circ) = \sqrt{3}, \text{ so } h = r\sqrt{3} = 8\sqrt{3}$$

$$\text{Volume} = \frac{1}{3} \pi r^2 h = \frac{\pi \cdot 8^2 \cdot 8\sqrt{3}}{3} = \frac{512\pi}{\sqrt{3}}$$

$$13. x = 2 \cos\left(\frac{5\pi}{6}\right) = -\sqrt{3}, y = 2 \sin\left(\frac{5\pi}{6}\right) = 1.$$



14. $r = 5, \phi = \frac{3\pi}{2}$



15. This a circle with center at the origin and radius 5.

