Math 75A

Final Exam

December 12, 2005

- No calculators, notes, or books are allowed.

- Please turn off your cell phones.

- You have 2 hours to complete the exam.

- The final exam counts as 20 percent of your grade.

- You may write on this test. When you are done please turn in your test and scantron form. The test and answers will be posted on the course web page on Wednesday.

1. Which of the following points lies on the graph of $f(x) = x^2 - 4x + 1$?

   (a) $(-1, 3)$
   (b) $(0, -4)$
   (c) $(1, -2)$
   (d) $(2, 2)$
   (e) $(3, -5)$

2. Which of the following is an equation of the line with slope 5 that passes through the point $(1, -3)$?

   (a) $y = 2x + 5$
   (b) $y = -8x + 5$
   (c) $y = 2x - 5$
   (d) $y = 5x - 2$
   (e) $y = 5x - 8$

3. Which of the following is the domain of the function $f(x) = \sqrt{x^2 - 4}$?

   (a) $(2, \infty)$
   (b) $[2, \infty)$
   (c) $(-2, 2)$
   (d) $(-\infty, -2] \cup [2, \infty)$
   (e) $(-\infty, 2) \cup (2, \infty)$

4. Find the distance between $(5, -3)$ and $(-2, 4)$.

    (a) 7

    (b) $7\sqrt{2}$

    (c) $\sqrt{10}$

    (d) 4

    (e) 14

5. Let $f = x^2 - 5$ and $g(x) = 2x + 3$. Find $(f \circ g)(x)$.

    (a) $2x^3 + 3x^2 - 10x - 15$

    (b) $2x^2 - 2$

    (c) $4x^2 + 14$

    (d) $2x^2 - 7$

    (e) $4x^2 + 12x + 4$

6. The graph of $f(x) = e^x$ is shifted 3 units to the right. Find an equation of the new curve.

    (a) $y = e^{x+3}$

    (b) $y = e^{x-3}$

    (c) $y = e^x + 3$

    (d) $y = e^x - 3$

    (e) $y = 3e^x$

7. Simplify the expression: $\dfrac{3 + \frac{2x+1}{x}}{\frac{5-x}{x}} + 2$

    (a) $\dfrac{3x + 11}{5 - x}$

    (b) $\dfrac{5 - x}{2x + 4}$

    (c) $\dfrac{5x + 3}{5 - x}$

    (d) $\dfrac{1 - 3x}{x^2}$

    (e) None of the above

8. Find the inverse function of $f(x) = \dfrac{x + 2}{x}$

    (a) $f^{-1}(x) = -\dfrac{x + 2}{x}$

    (b) $f^{-1}(x) = \dfrac{x}{x + 2}$

    (c) $f^{-1}(x) = \dfrac{2}{x - 1}$

    (d) $f^{-1}(x) = \dfrac{2}{x + 1}$

    (e) None of the above

9. Evaluate the expression: $\log_4 32 - \log_4 8$

    (a) 0

    (b) 1

    (c) 2

    (d) 3

    (e) None of the above

10. Solve the equation: $3^{x+4} = \dfrac{1}{9}$

    (a) -6

    (b) -4.5

    (c) -3.5

    (d) -2

    (e) None of the above

11. Evaluate the limit: $\displaystyle\lim_{x\to 3} \frac{x^2 - 9}{x^2 - 2x - 3}$.

    (a) 0

    (b) 1

    (c) 1.5

    (d) ∞

    (e) Does not exist

12. Find the vertical asymptote(s) of $f(x) = \dfrac{x^2 - 9}{x^2 - 2x - 3}$.

    (a) $x = -1$

    (b) $x = 1$

    (c) $x = 3$

    (d) $x = -1$ and $x = 3$

    (e) $x = -3$ and $x = 3$

13. Find the horizontal asymptote(s) of $f(x) = \dfrac{x^2 - 9}{x^2 - 2x - 3}$.

    (a) $y = -1$

    (b) $y = 1$

    (c) $y = 3$

    (d) $y = -1$ and $y = 1$

    (e) $y = 1$ and $y = 3$

14. Evaluate the limit: $\displaystyle\lim_{x \to 5^-} \dfrac{x^2 - 5}{x^2 - 6x + 5}$

    (a) 0

    (b) $\dfrac{1}{3}$

    (c) $-\infty$

    (d) ∞

    (e) None of the above

15. Which of the following intervals contains a root of $x^3 + 3x + 1 = 20$?

    (a) $[-1, 0]$

    (b) $[0, 1]$

    (c) $[1, 2]$

    (d) $[2, 3]$

    (e) $[3, 4]$

16. A particle moves along a straight line with equation of motion $s(t) = \sin(\pi t)$. Find its average velocity over the time interval $[0, 0.5]$.

    (a) -1

    (b) 0

    (c) 1

    (d) 2

    (e) None of the above

17. Find the slope of the tangent line to the curve $y = \dfrac{4}{x^3}$ at $\left(-2, -\dfrac{1}{2}\right)$.

    (a) -0.75

    (b) $-.5$

    (c) 0

    (d) .5

    (e) 2

18. If $f(x) = \dfrac{e^2\sqrt{x}}{x^2}$, find $f'(x)$.

    (a) $\dfrac{-e^2 x^{-\frac{1}{2}}}{4x}$

    (b) $-\dfrac{3}{2}x^{-\frac{5}{2}}$

    (c) $-\dfrac{3e^2}{2x^2\sqrt{x}}$

    (d) $ex^{-\frac{5}{2}}(2x - 1.5e)$

    (e) None of the above

19. Find all values of x at which the tangent line to the curve $y = \dfrac{x^2 - 5}{x + 3}$ is horizontal.

    (a) -3

    (b) -1

    (c) 1

    (d) $-1, -5$

    (e) $-3, \sqrt{5}$

20. Convert $105°$ to radians:

    (a) $-\dfrac{7}{12}$

    (b) $\dfrac{7}{12}\pi$

    (c) $-\dfrac{105}{180}\pi$

    (d) $\dfrac{105}{180}$

    (e) None of the above

21. Find the derivative of $\sin(4x)$ at $x = \dfrac{\pi}{6}$.

    (a) $-2\sqrt{3}$

    (b) -2

    (c) $\dfrac{\sqrt{3}}{2}$

    (d) $4\sqrt{3}$

    (e) None of the above

22. Simplify the expression: $\sin x + \dfrac{\cot x - \sin^2 x \cot x}{(\sec x)^{-1}}$.

   (a) $\sin x$

   (b) $\cos x$

   (c) $\csc x$

   (d) $\sec x$

   (e) $\tan x$

23. If $y = \cos(x^2)$, find $\dfrac{dy}{dx}$.

   (a) $-\sin(x^2)$

   (b) $\sin(2x)$

   (c) $-\sin(x^2)\cos(2x)$

   (d) $-\sin(x^2) + 2x\cos(x^2)$

   (e) $-2x\sin(x^2)$

24. If $f(x) = 5e^{2x-3}$, find $f'(4)$.

   (a) $2e^5$

   (b) $5e^5$

   (c) $10e^5$

   (d) $5\ln(e)e^5$

   (e) None of the above

25. Use implicit differentiation to find $y'(x)$ if $xy^3 - xy = 4x^2 + 6$.

(a) $\dfrac{8x - y^3 + y}{3xy^2 - x}$

(b) $\dfrac{8x}{3(y(x))^2 - 1}$

(c) $\dfrac{y - y^3}{3xy + x}$
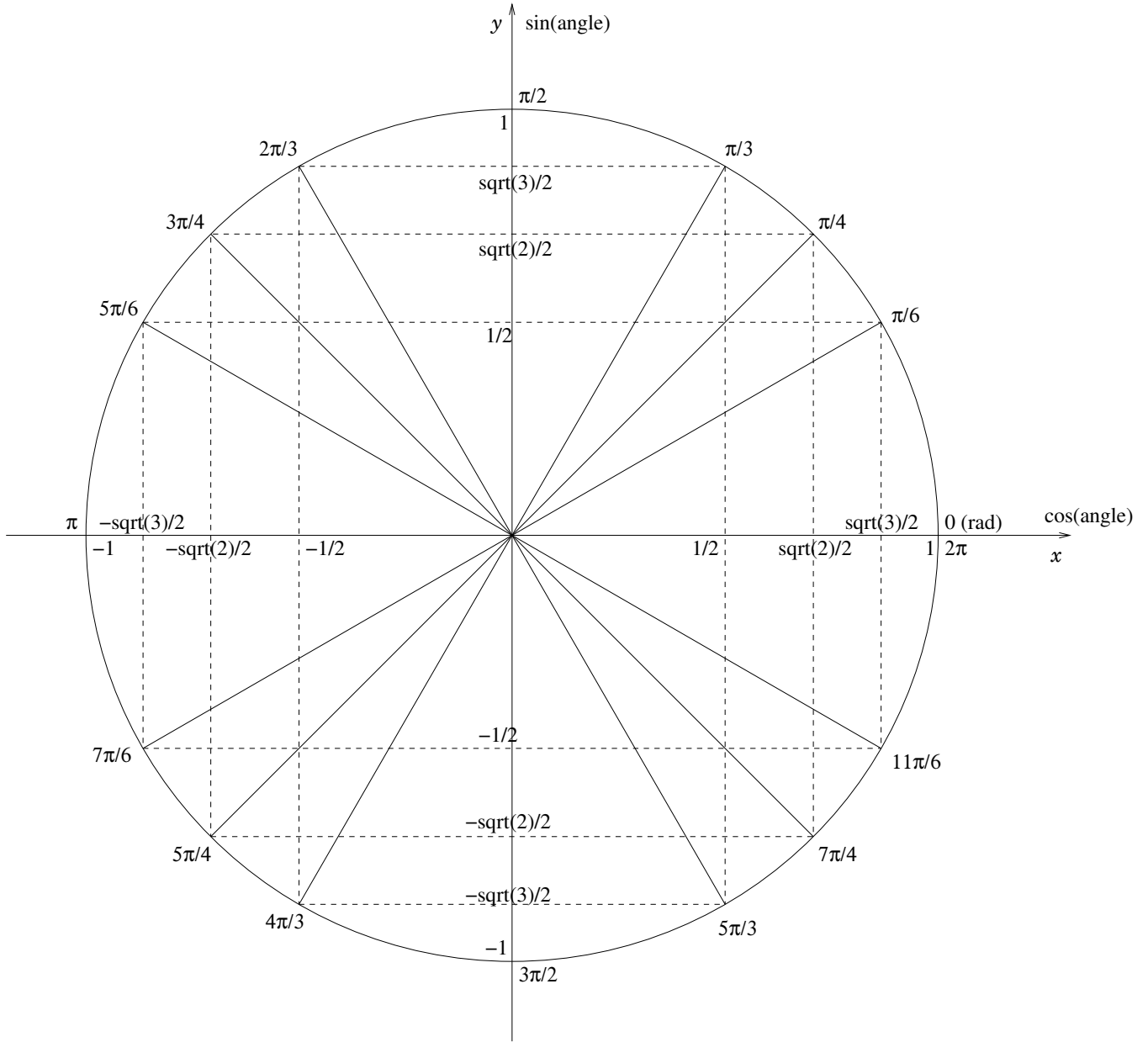
(d) $\dfrac{3xy^2 + y^3 - y - 8x}{x}$

(e) None of the above

Please double check all your answers before turning them in. Do not leave any questions unanswered – there is no penalty for guessing!

Good luck on your other final exams, and have a great winter break!

Answer key:

1c, 2e, 3d, 4b, 5e, 6b, 7a, 8c, 9b, 10a, 11c, 12a, 13b, 14c, 15d, 16d, 17a, 18c, 19d, 20b, 21b, 22c, 23e, 24c, 25a

y sin(angle)

π/2
1

2π/3   π/3

sqrt(3)/2

3π/4   π/4

sqrt(2)/2

5π/6   π/6

1/2

π   −sqrt(3)/2   sqrt(3)/2   0 (rad)   cos(angle)
−1   −sqrt(2)/2   −1/2   1/2   sqrt(2)/2   1 2π   x

−1/2

7π/6   11π/6

−sqrt(2)/2

5π/4   7π/4

−sqrt(3)/2

4π/3   5π/3

−1

3π/2

$$\cos(\alpha) = x \qquad\qquad \sec(\alpha) = \frac{1}{\cos(\alpha)} = \frac{1}{x}$$

$$\sin(\alpha) = y \qquad\qquad \csc(\alpha) = \frac{1}{\sin(\alpha)} = \frac{1}{y}$$

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{y}{x} \qquad\qquad \cot(\alpha) = \frac{1}{\tan(\alpha)} = \frac{\cos(\alpha)}{\sin(\alpha)} = \frac{x}{y}$$

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