

Math 75A

Final Exam

December 14, 2005

- No calculators, notes, or books are allowed.
- Please turn off your cell phones.
- You have 2 hours to complete the exam.
- The final exam counts as 20 percent of your grade.
- You may write on this test. When you are done please turn in your scantron form. You may keep your copy of the test. Both final exams (Monday and Wednesday) and answers will be posted on the course web page later today.

1. The graph of which of the following functions passes through the point $(2, 4)$?

(a) $f(x) = -x^2$

(b) $f(x) = x^2 + x$

(c) $f(x) = x^3 + 1$

(d) $f(x) = x - 2$

(e) $f(x) = 6 - x$

2. Which of the following is an equation of the line that passes through $(1, -3)$ and $(-1, -7)$?

(a) $y = 2x - 5$

(b) $y = 5x - 2$

(c) $y = -2x - 1$

(d) $y = -2x - 5$

(e) $y = -2x - 9$

3. Which of the following is the domain of the function $f(x) = \frac{1}{\sqrt{x-3}}$?

(a) $(0, \infty)$

(b) $[0, \infty)$

(c) $(3, \infty)$

(d) $[3, \infty)$

(e) $(-\infty, 0) \cup (0, \infty)$

4. Find the distance between $(-5, -9)$ and $(1, -1)$.

- (a) 7
- (b) $\sqrt{14}$
- (c) 10
- (d) 14
- (e) 100

5. Let $f = x^2 + 2$ and $g(x) = 7x - 8$. Find $(f \circ g)(1)$.

- (a) -3
- (b) 1
- (c) 3
- (d) 8
- (e) 13

6. The graph of $f(x) = x^2$ is shifted 3 units to the left and 1 unit upward. Find an equation of the new curve.

- (a) $y = x^2 - 3 + 1$
- (b) $y = (x - 3)^2 + 1$
- (c) $y = (x + 3)^2 + 1$
- (d) $y = (x + 1)^2 - 3$
- (e) $y = (x + 1)^2 + 3$

7. Simplify the expression: $\frac{\frac{1}{x+3} - \frac{1}{x+5}}{\frac{x}{x+3} + \frac{1}{x+5}}$

(a) $-\frac{8}{x^2 + 6x + 3}$

(b) $\frac{8}{x^3 + 6x^2 + 3x}$

(c) $\frac{x-1}{x^2+x}$

(d) $\frac{-x^3 - 12x^2 - 50x - 75}{x^3 + 6x^2 + 3x}$

(e) None of the above

8. Find the inverse function of $f(x) = \sqrt[3]{x+2} - 1$

(a) $f^{-1}(x) = (x+2)^3 - 1$

(b) $f^{-1}(x) = (x-2)^3 + 1$

(c) $f^{-1}(x) = (x+1)^3 - 2$

(d) $f^{-1}(x) = (x-1)^3 + 2$

(e) None of the above

9. Evaluate the expression: $\sin\left(\frac{7\pi}{6}\right)\tan\left(-\frac{\pi}{3}\right)$

(a) $2\sqrt{3}$

(b) $-\frac{2}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{2}$

(d) $-\frac{1}{2\sqrt{3}}$

(e) None of the above

10. Solve the equation: $\log_2 \left(\frac{1}{x} \right) = 8^2$

- (a) $\frac{1}{8}$
- (b) $\frac{1}{6}$
- (c) 6
- (d) 8
- (e) None of the above

11. Evaluate the limit: $\lim_{x \rightarrow 3} \frac{x^2 - 1}{x^2 - 2x - 3}$.

- (a) 0
- (b) 2
- (c) $-\infty$
- (d) ∞
- (e) Does not exist

12. Find the vertical asymptote(s) of $f(x) = \frac{x^2 - 1}{x^2 - 2x - 3}$.

- (a) $x = -1$
- (b) $x = 1$
- (c) $x = 3$
- (d) $x = -1$ and $x = 1$
- (e) $x = -1$ and $x = 3$

13. Find the horizontal asymptote(s) of $f(x) = \frac{x^2 - 1}{x^2 - 2x - 3}$.

- (a) $y = -1$
- (b) $y = 1$
- (c) $y = 3$
- (d) $y = -1$ and $y = 1$
- (e) $y = -1$ and $y = 3$

14. Evaluate the limit: $\lim_{x \rightarrow 25} \frac{x - 25}{\sqrt{x} - 5}$

- (a) 0
- (b) 1
- (c) 10
- (d) ∞
- (e) None of the above

15. Which of the following intervals contains a root of $x^3 + 3x + 10 = 0$?

- (a) $[-2, -1]$
- (b) $[-1, 0]$
- (c) $[0, 1]$
- (d) $[1, 2]$
- (e) $[2, 3]$

16. A particle moves along a straight line with equation of motion $s(t) = x^4 + 3x^2 - 2$. Find its acceleration at $t = 2$.

- (a) 1
- (b) 28
- (c) 44
- (d) 54
- (e) None of the above

17. Find the value of a for which the function $f(x) = \begin{cases} \sqrt{x} + 4 & \text{if } x < 4 \\ 3x + a & \text{if } x \geq 4 \end{cases}$ is continuous at $x = 4$.

- (a) -6
- (b) $\frac{1}{36}$
- (c) $\frac{1}{9}$
- (d) $\frac{1}{6}$
- (e) 6

18. If $f(x) = \frac{4x - 2}{x^2\sqrt{x}}$, find $f'(4)$.

- (a) -223
- (b) $\frac{1}{5}$
- (c) $\frac{3}{16}$
- (d) $-\frac{19}{128}$
- (e) None of the above

19. Find all values of x at which the tangent line to the curve $y = x^2 + 3x + 5$ has slope 7.

- (a) -1
- (b) -2
- (c) 2
- (d) -1 and -2
- (e) -2 and 2

20. Convert 12° to radians:

- (a) $\frac{1}{30}$
- (b) $\frac{1}{15}\pi$
- (c) $-\frac{12}{180}\pi$
- (d) $\frac{12}{180}$
- (e) None of the above

21. If $y = \cos^2(x)$, find $f\left(\frac{\pi}{4}\right)$.

- (a) -1
- (b) 1
- (c) $-\frac{1}{2}$
- (d) $\frac{1}{2}$
- (e) None of the above

22. If the cost function for a certain product is $C(x) = 60 + 2x + 0.05x^2 - 0.0005x^3$, find the marginal cost at the production level $x = 20$.

- (a) -2
- (b) 1.4
- (c) 16
- (d) 80
- (e) None of the above

23. If $y = \frac{2 + \tan x}{\sec x}$, find $\frac{dy}{dx}$.

- (a) $\frac{\sec x}{\tan x}$
- (b) $\cos x - 2 \sin x$
- (c) $\csc x$
- (d) $2 \sin x - \cos x$
- (e) $\frac{2 \cos x + 1}{\sin x}$

24. Find the derivative of $y = \frac{e^{2x} + 1}{e^x + 1}$ at $x = 0$.

- (a) $\frac{1}{4}$
- (b) $\frac{1}{2}$
- (c) 1
- (d) 3
- (e) None of the above

25. Use implicit differentiation to find y' if $x^3y + y^4 - x^2 = 6$.

(a) $3x^2y + 4y^3 - 2x$

(b) $\frac{2}{x^2} - \frac{3y}{x} - \frac{4y^3}{x^3}$

(c) $\frac{2x - 3x^2y}{4y^3}$

(d) $\frac{2x - 3x^2y}{x^3 + 4y^3}$

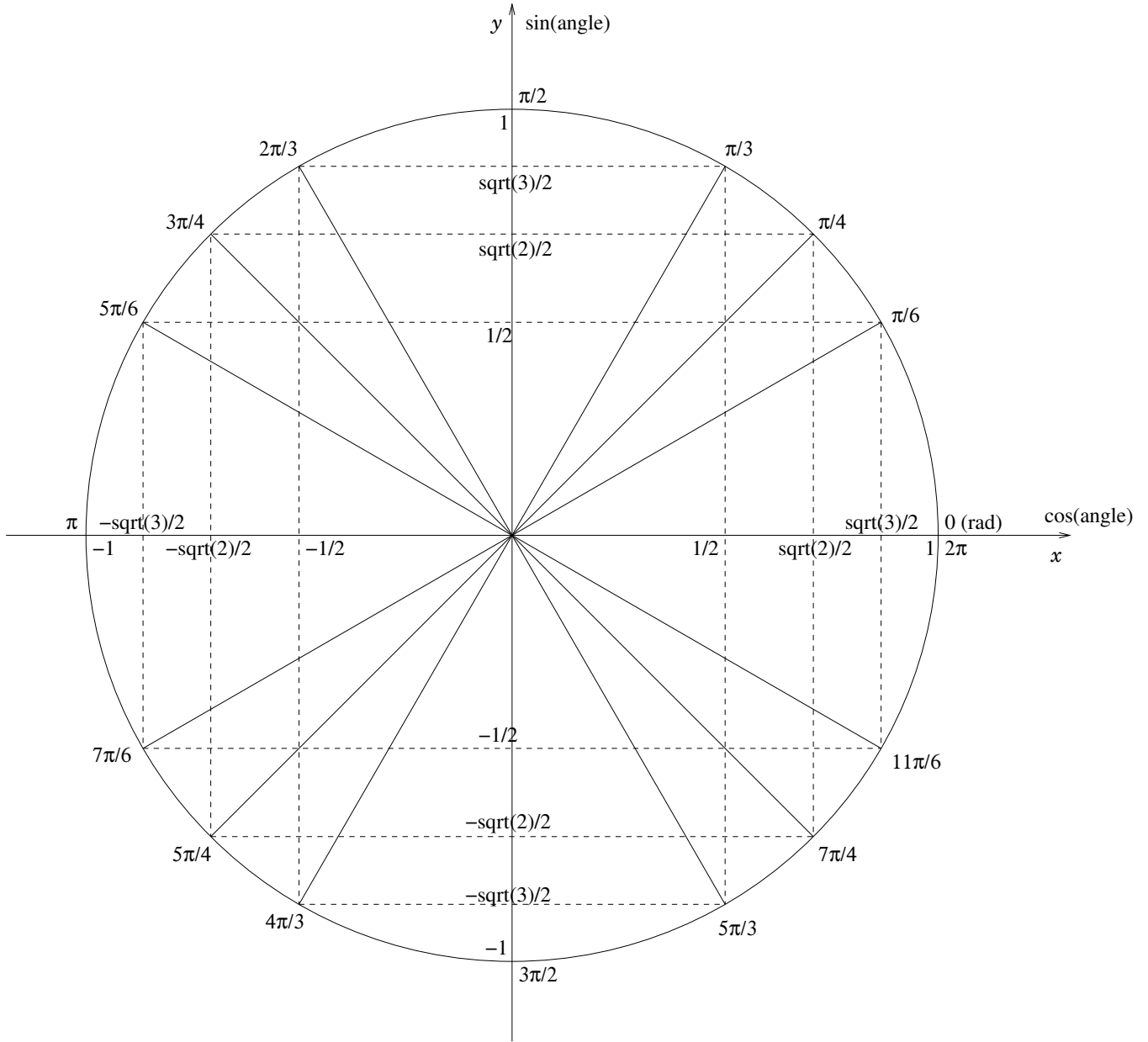
(e) None of the above

Please double check all your answers before turning them in. Do not leave any questions unanswered – there is no penalty for guessing!

Have a great winter break!

Answer key:

1e, 2a, 3c, 4c, 5c, 6c, 7e, 8c, 9c, 10b, 11e, 12c, 13b, 14c, 15a, 16d, 17a, 18d, 19c, 20b, 21a, 22e, 23b, 24b, 25d



$$\cos(\alpha) = x$$

$$\sec(\alpha) = \frac{1}{\cos(\alpha)} = \frac{1}{x}$$

$$\sin(\alpha) = y$$

$$\csc(\alpha) = \frac{1}{\sin(\alpha)} = \frac{1}{y}$$

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{y}{x}$$

$$\cot(\alpha) = \frac{1}{\tan(\alpha)} = \frac{\cos(\alpha)}{\sin(\alpha)} = \frac{x}{y}$$