Math 75A **Maria Nogin**

Review - 1

THEORY

Useful formulas

$$(a+b)^2 = a^2 + 2ab + b^2, \quad (a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)(a-b) = a^2 - b^2$$

Intervals

- $x \in (a,b) \iff a < x < b$
- $x \in [a,b) \iff a \le x < b$
- $x \in (a,b] \iff a < x \le b$
- $x \in [a,b] \iff a \le x \le b$
- $x \in (a,+\infty) \iff a < x$
- $x \in [a,+\infty) \iff a \le x$
- $x \in (-\infty,b) \iff x < b$
- $x \in (-\infty,b] \iff x \le b$

Absolute value

$$|x| = \begin{cases} x & \text{if} \quad x \ge 0 \\ -x & \text{if} \quad x < 0 \end{cases}$$

Fractions

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{ad}, \qquad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \qquad \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}, \qquad \frac{a}{b} = \frac{ac}{bc}$$

Distance formula

The distance between points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Lines

The slope of the line that passes through points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}.$$

The slope of a horizontal line is equal to 0.
The slope of a vertical line is undefined.
An equation of the line that passes through the point $P(x_1, y_1)$ and has slope m is

$$y - y_1 = m(x - x_1) \quad \text{(point-slope equation)}$$

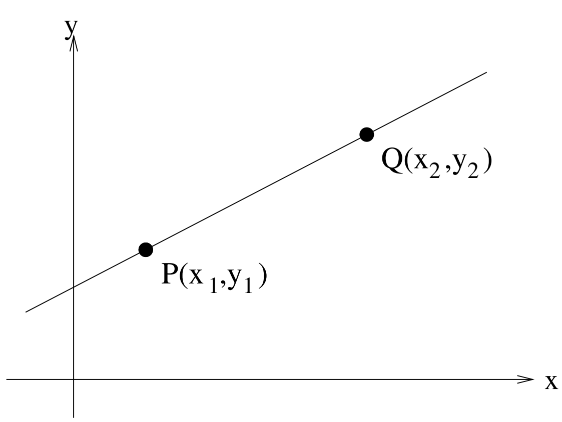
To find an equation of the line that passes through points $P(x_1, y_1)$ and $Q(x_2, y_2)$, first find its slope and then use the point-slope equation.

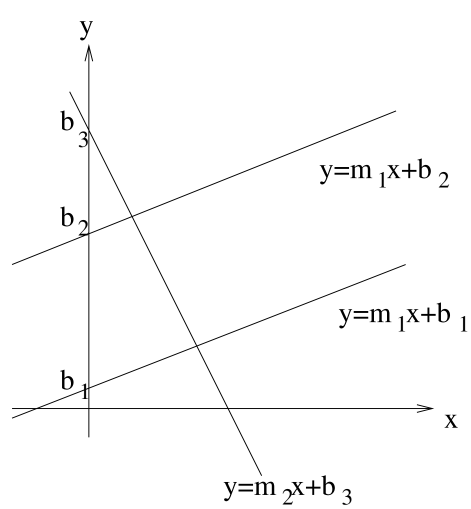
An equation of the line that has slope m and intersects the y-axis at the point $(0, b)$ is

$$y = mx + b \quad \text{(slope-intercept equation)}$$

Two non-vertical lines are parallel if and only if they have the same slope.
Two non-vertical lines with slopes m_1 and m_2 are perpendicular if and only if

$$m_1 \cdot m_2 = -1.$$





Circles

An equation of a circle with center at (a, b) and radius r can be written in the form

$$(x - a)^2 + (y - b)^2 = r^2.$$

Domain and range of a function

The **domain** of $f(x)$ is the set of all values of x for which $f(x)$ is defined.
The **range** of $f(x)$ is the set of all values of $y = f(x)$.

Important classes of functions

- Constant: $f(x) = a$
- Linear: $f(x) = mx + b$
- Quadratic: $f(x) = ax^2 + bx + c$
- Cubic: $f(x) = ax^3 + bx^2 + cx + d$
- Polynomial: $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$
- Rational: $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials
- Power: $f(x) = x^n$
- Root: $f(x) = \sqrt[n]{x}$
- Trigonometric: $f(x) = \sin x$, $\cos x$, $\tan x$, $\ldots$
- Exponential: $f(x) = a^x$
- Logarithmic: $f(x) = \log_a x$

See section 1.2 for **GRAPHS** of the above classes of functions.

Laws of Exponents

- $a^{x+y} = a^x \cdot a^y$
- $a^{x-y} = \dfrac{a^x}{a^y}$
- $(a^x)^y = (a^y)^x = a^{xy}$
- $\sqrt[x]{a} = a^{1/x}$
- $a^{-x} = \dfrac{1}{a^x}$
- $a^0 = 1$
- $a^1 = a$

The number e is approximately equal to 2.718281828459045. The function e^x is called the natural exponential function.

Trigonometry

$$180^{\mathrm{O}} = \pi \text{ rad} \qquad\qquad 1^{\mathrm{O}} = \frac{\pi}{180} \text{ rad}$$

$$\sin^2 x + \cos^2 x = 1, \quad \tan x = \frac{\sin x}{\cos x}, \quad \sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}$$

$$\sin(-x) = -\sin x, \qquad \cos(-x) = \cos x, \qquad \tan(-x) = -\tan x$$

$$\sin(x + 2\pi) = \sin x, \quad \cos(x + 2\pi) = \cos x, \quad \tan(x + \pi) = \tan x$$

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π
$\cos x$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1
$\sin x$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0

$$\arcsin x = \sin^{-1} x = y \;\text{ s.t. }\; \sin y = x \text{ and } -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

$$\arccos x = \cos^{-1} x = y \;\text{ s.t. }\; \cos y = x \text{ and } 0 \le y \le \pi$$

$$\arctan x = \tan^{-1} x = y \;\text{ s.t. }\; \tan y = x \text{ and } -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

Combinations of functions

$$(f \pm g)(x) = f(x) \pm g(x), \qquad (fg)(x) = f(x)g(x), \qquad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Composition of functions

$$(f \circ g)(x) = f(g(x))$$

Certain properties of functions

A function $f(x)$ is called **even** if $f(-x) = f(x)$ for all x in the domain of f. The graph of an even function is symmetric about the y-axis.

A function $f(x)$ is called **odd** if $f(-x) = -f(x)$ for all x in the domain of f. The graph of an odd function is symmetric about the origin.

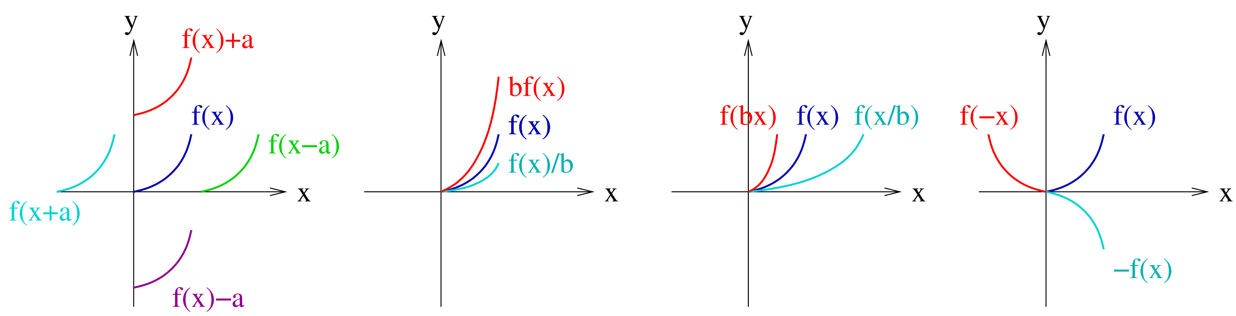
A function $f(x)$ is called **one-to-one** if for any value of y there exists at most one value of x such that $f(x) = y$. Equivalently, $f(x)$ is one-to-one if $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

$f(x)$ is not one-to-one if there exist $x_1 \neq x_2$ such that $f(x_1) = f(x_2)$.

Transformations of functions

Let $a > 0$ and $b > 1$. To obtain the graph of

- $y = f(x) + a$, shift the graph of $y = f(x)$ a units upward.
- $y = f(x) - a$, shift the graph of $y = f(x)$ a units downward.
- $y = f(x + a)$, shift the graph of $y = f(x)$ a units to the left.
- $y = f(x - a)$, shift the graph of $y = f(x)$ a units to the right.
- $y = bf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of b.
- $y = \dfrac{f(x)}{b}$, compress the graph of $y = f(x)$ vertically by a factor of b.
- $y = f(bx)$, compress the graph of $y = f(x)$ horizontally by a factor of b.
- $y = f\left(\dfrac{x}{b}\right)$, stretch the graph of $y = f(x)$ horizontally by a factor of b.
- $y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis.
- $y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis.



Inverse functions

If $f : A \to B$ is a one-to-one function then its inverse is a function $f^{-1} : B \to A$ defined by

$$f^{-1}(y) = x \qquad \text{if and only if} \qquad f(x) = y.$$

Equivalently, $f(x)$ and $g(x)$ are inverses of each other if $f(g(x)) = x$ and $g(f(x)) = x$ for all x.

To find the inverse of a one-to-one function, use the following procedure:

1. Write $f(x) = y$.
2. Solve the above equation for x.
3. You obtain an equation of the form "$x =$ a function of y". The function on the right is $f^{-1}(y)$.
4. If you wish to have the answer as a function of x rather than a function of y, replace each y in step 3 by x. You'll get $f^{-1}(x)$.

Graphs $y = f(x)$ and $y = f^{-1}(x)$ are symmetric about the line $y = x$.

The **logarithmic function** $\log_a x$ is defined as the inverse of the function e^x.
The function $\log_e x$ is called the natural logarithmic function and is denoted by $\ln x$.

Laws of logarithms

- $\log_a(xy) = \log_a x + \log_a b$
- $\log_a \left(\dfrac{x}{y}\right) = \log_a x - \log_a b$
- $\log_a b \cdot \log_b c = \log_a c$
- $\log_a x = \dfrac{\ln x}{\ln a}$
- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a(a^x) = x$
- $a^{\log_a x} = x$
- $\ln(e^x) = x$
- $e^{\ln x} = x$