Math 75A   **Maria Nogin**
Practice test 3

The actual exam will consist of 6 multiple choice questions and 6 regular problems.
You will have 50 minutes to complete the exam.

Multiple choice questions: circle the correct answer

1. Find the derivative of $\sqrt{2x}$.

   A. $\dfrac{2}{\sqrt{x}}$ **B.** $\dfrac{2}{\sqrt{2x}}$ **C.** $\dfrac{1}{2\sqrt{x}}$ **D.** $\dfrac{1}{\sqrt{2x}}$ **E.** $\dfrac{1}{2\sqrt{2x}}$

2. Evaluate the limit: $\displaystyle\lim_{x\to 0}\dfrac{\sin(3x)}{5x}$

   A. 0   **B.** 0.6   **C.** $\frac{1}{5}$ **D.** $\frac{5}{3}$ **E.** Does not exist

3. Simplify the expression: $\dfrac{8x^3\sqrt{x}}{(3x^2)^2 + 7x^4}$

   A. $\dfrac{8\sqrt{x}}{10x^2}$ **B.** $\dfrac{\sqrt{x}}{2}$ **C.** $\dfrac{1}{2\sqrt{x}}$ **D.** $\dfrac{4}{5\sqrt{x}}$ **E.** $4\sqrt{x}$

4. The position of an object at time t is given by $s(t) = 4\sin(t) + 2\cos(t)$. Find the velocity of this object at $t = \dfrac{\pi}{3}$.

   A. $1 + \sqrt{3}$ **B.** $1 + 2\sqrt{3}$ **C.** $1 - 2\sqrt{3}$ **D.** $2 + \sqrt{3}$ **E.** $2 - \sqrt{3}$

5. Find the equation of the line tangent to the curve $y = x^2 + 4x + 4$ at $(1, 9)$.

   A. $y = 9x$ **B.** $y = 6x - 15$ **C.** $y = 6x + 3$ **D.** $y = 2x + 1$
   E. None of the above

6. If $f(3) = 2$, $f'(3) = 4$, $g(3) = 5$, and $g'(3) = 6$, then the derivative of $\dfrac{f(x)}{g(x)}$ at $x = 3$ is $\left(\dfrac{f}{g}\right)'(3) =$

   A. 0.32   **B.** 2/3   **C.** $-8/25$ **D.** 0   **E.** Undefined

7. If $f(x) = 4^{3x}$, find $f'(x)$.

   A. 4^{3x} **B.** $3 \cdot 4^{3x}$ **C.** 12^{3x} **D.** $\ln(4)4^{3x}$ **E.** $3\ln(4)4^{3x}$

Regular problems: show all your work

8. Differentiate the following functions:

    (a) $f(x) = 7x - 3$

    (b) $p(s) = s^5 - 2s^4 + 3s^3 - 4s^2 + 5s - 6$

    (c) $f(t) = \dfrac{3t^2 - 5t + 1}{\sqrt{t}}$

    (d) $g(x) = x^2 - \dfrac{x^3}{\sqrt[4]{x}} + \dfrac{3}{x}$

    (e) $q(y) = \dfrac{y^2 + y + 1}{y + 1}$

    (f) $y = 3\sin(x^5) + \dfrac{\pi}{2}$

    (g) $f(x) = \cos(4)(x^3 - 3x)$

    (h) $g(x) = \dfrac{x^3 - 5}{\cos(-x)}$

    (i) $h(x) = \tan(x)\left(\dfrac{1}{\sqrt[4]{x^3}} + \dfrac{2}{x}\right)$

    (j) $f(t) = 5e^x - 8\cdot 3^x + 9x^2$

9. Find the points where the tangent line to the graph of $f(x) = x^5 - 80x$ is horizontal.

10. Find an equation of the tangent line to $y = \sqrt{2x + 3}$ at $(3, 3)$.

11. Evaluate the limits:

    (a) $\lim\limits_{x \to 0} \dfrac{\sin(6x)}{\sin(7x)}$

    (b) $\lim\limits_{x \to 0} \dfrac{2x}{\tan(4x)}$

    (c) $\lim\limits_{x \to 0} \tan(5x)\csc(x)$

12. Solve for $\dfrac{dy}{dx}$: $\quad 5x\left(8y\dfrac{dy}{dx} + x^2\right) = 7\dfrac{dy}{dx} - 3xy^3.$

13. Consider the curve given by $x^3y^3 - 3xy^3 + 4y = 6$.

    (a) Use implicit differentiation to find $y'(x)$.

    (b) Check that the point $(2, 1)$ lies on this curve.

    (c) What is the slope of the tangent line to this curve at $(2, 1)$?

14. A snowball is melting (so it is decreasing). Find the rate of decrease of its volume with respect to the radius when the radius is 3 cm. (Recall that the volume of a ball is $V = \frac{4}{3}\pi r^3$.)